Math 111 EXAM I, October 2, 2002

Read each problem carefully. Show all your work for each problem! Use only those methods discussed thus far in class. No Calculators!

1. (15) Evaluate the following limits:

$$(a)\ \lim_{x\to 2}\sqrt{\frac{2x^2+1}{2x}}, \qquad (b)\ \lim_{\theta\to 0}\frac{\tan(\pi\theta)}{\theta}, \qquad (c)\ \lim_{t\to -3}\frac{t^2+4t+3}{t^2-9}.$$

2. (15) Find the following for the function $f(t) = t^4 - 2t^3 + 4$:

   (a) The instantaneous rate of change of f with respect to t,

   (b) The average rate of change of f over the interval $-1 \le t \le 2$,

   (c) The equation of the tangent line to f at $t = 2$.

3. (10) Find the constants c and d so that the function given below is continuous for all x:

$$f(x) = \begin{cases} x^2+3, & x < 2 \\ c, & x = 2 \\ cx+d, & x > 2. \end{cases}$$

4. (15) Find dy/dx using the differentiation rules discussed in class (do not simplify):

$$(a)\ y = \frac{x^3-4x+5}{x^2+9}, \qquad (b)\ y = (7x^2+3)^4(5x-7), \qquad (c)\ y = \frac{1}{x(x^3+1)^{3/2}}.$$

5. (10) Find the maximum and minimum values attained by the given functions on the indicated closed intervals:

$$(a)\ f(x) = 3x^{1/2} - x^{3/2},\ x \in [0,4], \qquad (b)\ f(x) = x^2 + 2x + 1,\ x \in [-1,1].$$

6. (15) Evaluate the following limits:

$$(a)\ \lim_{x\to 0}\cot(x), \qquad (b)\ \lim_{x\to 1^-}\frac{|1-x|}{1-x^2}, \qquad (c)\ \lim_{x\to 0}\frac{\sqrt{1+x}-1}{\sin(x)},.$$

7. (10) A ball is thrown straight up from a height y_0 ft above ground with an initial velocity of 16 ft/s and reaches a maximum height of 64 ft. What was the initial height, y_0? (Assume $g = 32$ ft/s^2, and recall $y = -\frac{1}{2}gt^2 + v_0t + y_0$).

8. (10) Use the definition of the derivative to show that

$$\frac{d}{dx}(xf(x)) = xf'(x) + f(x).$$