Math 111 EXAM II, November 6, 2002

Read each problem carefully. Show all your work for each problem! No Calculators!

1. (10) Find the linear approximation of the function $f(x) = \sin(x)$ about the point $a = 0$, and then about the point $a = \pi/6$.

2. (15) Find dy/dx for the following:

$$(a)\ \ y = \tan^2(x)\sec(x^2), \qquad (b)\ \ y = \cos(\sin(\sqrt{x})), \qquad (c)\ \ y^2 = 3x - x\sin(y).$$

3. (15) A rectangular poster is to contain 50 in^2 of printed matter, with 4 in. margins at the top and bottom, and 2 in. margins on each side. What are the dimensions of the poster which will require the least amount of paper?

4. (15) Show that the function $f(x) = x^3 - 10x + 5$ has a root in the interval [0,1]. Find an approximation to this root using Newton's Method. Use $x_0 = 0$ and calculate the first two iterations, i.e. x_1 and x_2.

5. (10) Find all horizontal, vertical and slant asymptotes for the following functions:

$$(a)\ \ y = \frac{x^2+1}{x^2+7x}, \qquad (b)\ \ y = \frac{1-x^3}{x^2-6x}.$$

6. (10) A 20 ft. ladder is leaning against a building. If the bottom of the ladder is sliding along the level pavement directly away from the building at 1 ft/s, how fast is the top of the ladder moving down when the foot of the ladder is 5 ft from the wall?

7. (15) For the function below, find all of the following if they exist: (i) all local extrema, (ii) points of inflection, (iii) intervals where the function is increasing or decreasing, (iv) intervals of upward and downward concavity, and (v) all asymptotes. Also, sketch a plot of the curve $y = f(x)$.

$$f(x) = \frac{x^2}{4-x^2}.$$

8. (10) A ball is thrown straight up from the ground into the air with an initial velocity of 52 ft/s. The ball is observed by a man standing on the ground 30 ft. from the spot directly beneath the ball. At what rate is the distance between the man and the ball changing after 2 s? Recall $y = -\frac{1}{2}gt^2 + v_0t + y_0$, and use $g = 32ft/s^2$.