

Math 111 FINAL EXAM, May 9, 2003

Read each problem carefully. Show all your work for each problem! No Calculators!

1. (12) Evaluate the following limits:

$$(a) \lim_{x \rightarrow -2} \frac{x+2}{x^2+4}, \quad (b) \lim_{\theta \rightarrow 0} \frac{\theta}{\tan(\pi\theta)}, \quad (c) \lim_{t \rightarrow -3} \frac{t^2+6t+9}{t^2-9}, \quad (d) \lim_{x \rightarrow 5^-} \frac{x-5}{|x-5|}.$$

2. (8) Find two positive real numbers such that their sum is 50 and their product is as large as possible.

3. (8) Find the area of the region bounded by the curves $y = x^2$ and $y = 2x$.

4. (12) Calculate dy/dx for the following:

$$(a) y = \frac{6x+2}{3x-4}; \quad (b) y = \sqrt{x+\sqrt{x}}; \quad (c) xy = \tan(xy); \quad (d) y = \int_2^{3x} \sin(t^2) dt.$$

5. (8) Apply the h -definition of the derivative to find $f'(x)$ for the function $f(x) = 1/x$.

6. (8) Calculate the trapezoidal approximation, with $n = 4$, to the integral $\int_1^3 x^2 dx$.

7. (14) Evaluate the following integrals (a,b,c: 3 points; d: 5 points):

$$(a) \int (2 + \frac{1}{x^2}) dx; \quad (b) \int x\sqrt{9+x^2} dx; \quad (c) \int_0^{\pi/4} \sin(x) \cos(x) dx;$$

$$(d) \int_0^1 \sqrt{1-\sqrt{x}} dx; \quad (\text{Hint: substitute } u = 1 - \sqrt{x}).$$

8. (10) For the function $f(x) = 2x + \frac{1}{x^2}$, find the following if they exist: (i) all local extrema, (ii) points of inflection, (iii) intervals where the function is increasing or decreasing, (iv) intervals of upward and downward concavity, and (v) all asymptotes. Also, sketch a plot of the curve $y = f(x)$.
9. (10) Find the volume of the solid that is generated by rotating the plane region bounded by the curves $y = 1 - x^2$ and $y = 0$ about the x -axis.
10. (10) The acceleration, $a(t)$, initial velocity, $v(0)$, and initial position, $x(0)$ of a particle for the time interval $t \in [0, 9]$ are given by

$$a(t) = \frac{6}{\sqrt{t}} - 6; \quad v(0) = 0, \quad x(0) = 0, \quad t \in [0, 9].$$

- (a) Find the particle's position function, $x(t)$.

- (b) What are the particle's maximum and minimum velocities on this interval?