Math 111 FINAL EXAM, December 13, 2002

Read each problem carefully. Show all your work for each problem! No Calculators!

1. (10) Apply the h-definition of the derivative to find $f'(x)$ for the function $f(x) = 1 + \sqrt{x}$.

2. (20) Evaluate the following limits:

$$(a)\ \lim_{t\to 2} \frac{t^3 - 3t - 1}{\sqrt{t^3+1}}, \quad (b)\ \lim_{x\to\infty} \frac{4 - x^2}{2x^2 + 6x - 8}, \quad (c)\ \lim_{x\to 1^-} \frac{\cos(\pi x)}{x^2 + 2x + 1}, \quad (d)\ \lim_{x\to 1^+} \frac{|x-1|}{x - x^2}.$$

3. (10) Find the area of the region bounded by the curves $y = 6 - x^2$ and $y = 3 - 2x$.

4. (20) Calculate dy/dx for the following:

$$(a)\ y = \frac{x^3 - 3x}{x^2 + 1}; \quad (b)\ y = \cos(\sqrt{x^2+1}); \quad (c)\ y = x\sin(xy); \quad (d)\ y = \int_{x^2}^{10} \sqrt{1+t^2}\, dt.$$

5. (10) Calculate the trapezoidal approximation, with $n = 4$, to the integral $\int_2^4 \frac{1}{x-1}\, dx$.

6. Suppose the velocity of a particle moving along a straight line is $v(t) = 6t^2 - 6t - 12$ m/s during the time interval [0,3].

   (a) (5) Find the acceleration of the particle.

   (b) (5) Find the average velocity of the particle over this interval.

   (c) (10) Find the total distance travelled by the particle.

7. (15) Evaluate the following integrals:

$$(a)\ \int_1^4 \frac{5x^2 - 3x}{\sqrt{x}}\, dx; \quad (b)\ \int \sqrt{\tan(x)}\sec^2(x)\, dx;$$

$$(c)\ \int_0^1 \sqrt{1-\sqrt{x}}\, dx; \ \text{(Hint: substitute } u = 1 - \sqrt{x}).$$

8. (15) For the function $f(x) = x - \frac{1}{x}$, find the following if they exist: (i) all local extrema, (ii) points of inflection, (iii) intervals where the function is increasing or decreasing, (iv) intervals of upward and downward concavity, and (v) all asymptotes. Also, sketch a plot of the curve $y = f(x)$.

9. (15) A solid has as its base the circular region in the xy-plane bounded by the curve $x^2 + y^2 = 1$. Find the volume of the solid if every cross section perpendicular to the x-axis is an equilateral triangle.

10. (15) Two sides of a triangle are 4 inches long. What should the angle between these sides, θ, be to maximize the area of the triangle?