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So $V(x,t) = \sum_{n=0}^{\infty} A_n \sin\left[\frac{(n+\frac{1}{2})\pi x}{L}\right] e^{-\frac{2(n+\frac{1}{2})^2\pi^2}{L^2}t}$

and it remains to satisfy the initial condition

$$V(x,0) = x = \sum_{n=0}^{\infty} A_n \sin\left(\frac{(n+\frac{1}{2})\pi x}{L}\right) \Rightarrow$$

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$$A_n = \frac{\int_0^L x \cdot \sin\left[\frac{(n+\frac{1}{2})\pi x}{L}\right] dx}{\underbrace{\int_0^L \sin^2\left[\frac{(n+\frac{1}{2})\pi x}{L}\right] dx}_{L/2}} = \frac{2}{L}\int_0^L x \sin\left[\frac{(n+\frac{1}{2})\pi x}{L}\right] dx$$

and $u(x,t) = V(x,t) + x + 10$

#2 Separate variables

$u(x,t) = \phi(x) \cdot h(t) \Rightarrow$

$$\frac{h'(t)}{h(t)} = \frac{\phi''(x)}{\phi(x)} - 1 \Rightarrow \frac{h'(t)}{h(t)} + 1 = \frac{\phi''(x)}{\phi(x)} = -\lambda \quad \text{so}$$

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$$\begin{cases} \phi''(x) + \lambda\phi(x) = 0 \,;\, 0 < x < L \\ \phi'(0) = 0 \\ \phi'(L) = 0 \end{cases} \quad \text{and} \quad h'(t) + (1+\lambda)h(t) = 0 \quad t > 0.$$

$\longrightarrow$ familiar cosine series