Math 630-102
Homework 3
Due date: February 8, 2007

Group work on h/w assignments is not allowed. No credit is given for results without a solution or an explanation. Late homework is not accepted.

Section 1.6

Problem 6. Use Gauss-Jordan method to invert A_1, A_2, A_3.

Note: For each of the three matrices, use your intermediate step [U | L^{-1}] to check that U = L^{-1}A (see note below if in doubt)

Problem 8. Show that *A* has no inverse by solving *Ax* = 0, and by failing to solve…

Section 2.1

Problem 1 b (we did part *a* in class) Construct a subset of the *x*-*y* plane R^2 that is…

There are infinitely many subsets of R^2 which are closed under multiplication but not under addition/subtraction. Give two examples, different from the one in the back of the book: describe one subset constructed from a single vector, and another example subset that contains two linearly independent vectors among its elements.

Problem 2. Which of the following subsets of R^3 are actually subspaces?

Problem 3. Describe the column space and the nullspaces of the matrices (Explain your answers!)

Note that for an $m \times n$ matrix A, the nullspace N(A) is a subset of R^n, since it is the subset of *solutions* of the system A *x*=0, and therefore, this space is composed of vectors with one component for each unknown. In contrast, the column space C(A) is a subspace of R^m – the number of components of its vectors is the number of rows (equations), which is *m*.

Problem 6. Let P be the plane in 3-space with equation x + 2y + z = 6…

Problem 8. Which of the following descriptions is correct? The solution *x* of…

Problem 9. Show that a set of nonsingular 2 by 2 matrices is not a vector space…

If you are confused how *matrices* could possibly form a *vector* space, just think of a 2 by 2 matrix as a collection of 4 numbers, so the most general (largest) vector space of

2 by 2 matrices is R^4. Importantly, both matrix addition/subtraction and scalar multiplication are well defined operations.

Problem 21. Describe the column spaces (lines or planes) of these particular matrices:

Problem 22. For which right-hand sides (find a condition on b_1, b_2, b_3) are these systems solvable?

Summary of Gauss-Jordan method:

Gauss-Jordan method is equivalent to the standard Gaussian elimination plus back-substitution applied to the augmented matrix of the system $A A^{-1} = I$, and therefore involves a series of row operations accomplishing these two standard steps:

$[A \mid I]$ → elimination (LU factorization) →$[U \mid L^{-1}]$ → backsubstitution → $[I \mid A^{-1}]$

Without resorting to augmented matrices, these two steps should be understood as two matrix multiplications applied to the original equation $AA^{-1} = I$:

$\mathbf{A}\, A^{-1} = \mathbf{I}$ → elimitation is equivalent to multiplication from the left by L^{-1}, so $L^{-1}AA^{-1}=\mathbf{UA^{-1}}$ (since A=LU); on the right we have $L^{-1} I = L^{-1}$, yielding:

$\mathbf{U}\, A^{-1} = \mathbf{L^{-1}}$ → backsubstitution step is equivalent to multiplying equation on the left by U^{-1}, so $U^{-1}UA^{-1} = \mathbf{I\, A^{-1}}$; on the right: $U^{-1}L^{-1}=(LU)^{-1}=\mathbf{A^{-1}}$**:**

$\mathbf{I}\, A^{-1} = \mathbf{A^{-1}}$ (final result)

Note that A^{-1} is the unknown in the first two equations, and emerges as the right-hand side in the last equation
Here I ignored the U→DU factorization step, since this step is the most obvious; therefore, in the above equations the pivots of U are not necessarily equal to 1