Math 630-102
Homework 4
Due date: February 15, 2007

Group work on h/w assignments is not allowed. No credit is given for results without a solution or an explanation. Late homework is not accepted.

Section 2.2

Problem 2. Reduce A and B to echelon form, to find their ranks. Which variables are free? Find the special solutions to $A x = 0$ and $B x = 0$. Find all solutions.

Note: repeat all steps done in class. **The "special solutions" are the null-space basis** (the term *basis* is introduced in the following section)**.** "Find all solutions" means to write the general null-space element generated by the N(A) basis.

Problem 5. Write the complete solutions $x = x_p + x_n$ to these systems, as in equation (4)

Problem 8. Under what conditions on b_1 *and* b_2 (if any) does $A x = b$ have a solution?

Note: The two vectors in the nullspace are the null-space basis. When solving $A x = b$, assume an arbitrary (variable) right-hand-side $\boldsymbol{b}$ with components b_1 and b_2

Section 2.3

Please read the whole book section carefully if the concept of linear independence is new to you (this is one of the simpler sections) – this concept is fundamental for the entire course.

Problem 1. Show that v_1, v_2, v_3 are independent but v_1, v_2, v_3, v_4 are dependent

Note: the last sentence explains how to answer the question posed in the first sentence.

Problem 2. Find the largest possible number of independent vectors among…

Note: The best way to do this is to carry out the row operations to obtain U and find the pivot columns

Problem 27. *U* comes from *A* by subtracting row 1 from row 3:… Find bases for the two column spaces. Find bases for the two row spaces. Find the bases for the two nullspaces.

Note that the row space basis is composed of rows containing the pivots. You have to find the three sets of bases for both *U* and *A*, although usually we are only concerned with the C(*A*) and N(*A*) of the original matrix *A*. So this question tests your understanding of the (possible) differences between the fundamental subspaces of *A* and *U*, respectively. Note also that the basis is never unique, so the fact that the bases are different does not necessarily mean that the subspaces are different.

Problem 30. By locating the pivots, find a basis for the column space of U=… Express each column that is not in the column space…

Section 2.4

Problem 18. Find a basis for each of the four subspaces of A = […] = […] [….] [*Matrix is already factored into L U for your convenience – V.M.*]

Note: we only covered three fundamental subspaces (C(A), N(A), and the row space C(A^T)) so far, so don't worry about the 4th one (the left null-space will be covered next time). The row space basis is composed of rows containing pivots.

Extra question for last problem. Verify that the product of the row space basis vectors (taken as *row* vectors) and each of the N(*A*) basis vectors is zero (orthogonality property)