Math 630-102
Homework 5
Due date: February 22, 2007

<u>Group work on h/w assignments is not allowed. No credit is given for results without a solution or an explanation. Late homework is not accepted.</u>

Note on calculating the left null-space: As the notation $N(A^T)$ indicates, the left null-space is simply the normal (right) null-space of the transposed matrix. As I mentioned in class, you cannot use U^T to find the left null-space, since row operations transform C(A) and $N(A^T)$, so even though N(A)=N(U), $N(A^T)$ is different from $N(U^T)$.

Section 2.4

Problem 18 (Problem 13 ISE): Please complete this problem by finding $N(A^T)$, by finding the (right) null-space of A^T (starting from the transpose of the original matrix, and performing the usual row operations). Check your answer by showing that the $N(A^T)$ basis is orthogonal to the C(A) basis.

Problem 2. Find the dimension and construct a basis for the four fundamental subspaces associated with each of the matrices A=[2×4] and U=[2×4]

Since U is obtained from A, this problem checks your understanding of the differences between the fundamental subspaces of A and U.

Problem 4. Describe the four subspaces in three-dimensional space associated with A=[3×3]

Here you have to describe the geometry of the subspaces, as we did in class. You don't have to draw them though.

Problem 10. If Ax=B always has at least one solution, show that the only solution to $A^Ty=0$ is $y=0$. *Hint:* what is the rank?

To solve this problem, recall the relationships between the dimensions of the fundamental subspaces.

Problem 14. Find a left-inverse and/or a right-inverse (when they exist) for…

Section 2.6

Note: the first couple of problems are not from the book

Problem I (not from the book). Show that a simple transformation called *translation* which adds a constant vector c to each vector ($x' = x + c$, where x, x' and c are in some R^n) is *not* a linear transformation (hint: this is trivial; just a check of the concept of *linear* transformation)

Problem II (not from the book). A transformation is length-preserving if, obviously, $||x'||=||x||$, where $x'=Ax$. What condition does the transformation matrix A has to satisfy in order for the transformation to be length-preserving? (Hint: recall the expression for $||x||^2$, and examine the condition $||x'||^2=||x||^2$). Which of the four basic transformations we learned in class (stretching/contraction, rotation, reflection, projection) are *always* length-preserving? Prove your answer.

Problem 1. What matrix has the effect of rotating every vector through 90 degrees and then projecting the result onto the x-axis?...

Hint: take matrix products

Problem 3. The matrix A=… produces a stretching in the x-direction. Draw the circle…

Section 3.1

Problem 3. Two lines in the plane are perpendicular when the product of their slopes is -1. Apply this to the vectors…

Problem 7. Find a vector x orthogonal to the row space of A, and a vector y orthogonal to the column space, and a vector z orthogonal to the nullspace…

Note: Recall the geometry of the four fundamental subspaces. Give just one example of each vector, not the whole orthogonal complement.

Problem 9. Find the orthogonal complement of the plane spanned by the vectors (1, 1, 2) and (1, 2, 3) by taking…

The geometry of the fundamental subspaces:

$Ax_N = 0 \rightarrow$ $C(A^T) \perp N(A)$ (orthogonal complements in R^n)
$\dim C(A^T) + \dim N(A) = n \rightarrow \dim N(A) = n - r$

$A^T y_N = 0 \rightarrow$ $C(A) \perp N(A^T)$ (orthogonal complements in R^m)
$\dim C(A) + \dim N(A^T) = m \rightarrow \dim N(A^T) = m - r$