Math 630-102
Homework #6
Due date: February 29, 2007

<u>Group work on h/w assignments is not allowed. No credit is given for results without a solution or an explanation. Late homework is not accepted.</u>

Section 2.6

Problem I (not in book) Show that integration of 3^{rd} degree polynomials (implemented as a 5×4 matrix, *S*) applied **after** the differentiation of 4^{th} degree polynomials (4×5 matrix, *D*) is **not** an identity transformation (*SD* does **not** equal *I*). Therefore, *S* is not a left inverse of *D*, and *D* is not a right inverse of *S*. Is it possible that *D* has a left inverse at all? Is it possible that *S* has a right inverse? Answer these questions in terms of the rank of these matrices (consult the summary on p. 108 if in doubt).

Problem 7. On the space P_3 of cubic polynomials, what matrix represents d^2/dt^2 ? [second derivative] ? Construct the 4-by-4 matrix…

Problem 8. From the cubics [cubic polynomials] P_3 to the fourth-degree polynomials P_4, what matrix represents multiplication by 2+3t ?

Section 3.2

Problem 21. Compute the projection matrices aa^T/a^Ta onto the lines through a_1=(-1,2,2) and a_2=(2,2,-1). Multiply those projections and explain why their product P_1P_2 is what it is.

Problem 22. Project b=(1,0,0) onto the lines through a_1 and a_2 in Problem 21 and also onto a_3=(2,-1,2). Add the three projections $p_1+p_2+p_3$.

Problem 23. Continuing Problems 21-22, find the projection matrix P_3 onto a_3=(2,-1,2). Verify that $P_1+P_2+P_3=I$.

Section 3.4

Repeat Exercise 5 (Gram-Schmidt) on page 180 ("Suppose the independent vectors are a, b, c…") for the following set of three vectors:

$$a = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \qquad c = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Check that the final vectors are orthonormal: place them into columns of some matrix Q, and verify that $Q^TQ=I$

.

Note: the vectors *a*, *b*, *c* are the three vectors that I called c_1, c_2, c_3 in class (columns of some matrix A). Vectors q_1, q_2 and q_3 were called b_1, b_2 and b_3 in class.