Math 630-102
Final Exam
May 3, 2007

This is a closed-book exam: notes and calculators are *not* allowed. Please explain your solutions to receive full credit. Check your answers.

1. (23) Consider the linear system Ax=b with A=$\begin{bmatrix} 2 & 1 & 1 & 0 \\ 2 & 2 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$, b=$\begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$

a) Find the **reduced** row-echelon form R of matrix A.
b) Find the **dimensions** of the four fundamental subspaces of A
c) Find the **general** solution to A x = b.
d) Does the system A x = b have a solution for **any** b?

2. (27) Consider a matrix A = $\begin{bmatrix} 1/4 & 1/2 \\ 3/4 & 1/2 \end{bmatrix}$

a) Diagonalize the matrix A

b) Solve the difference equation u_{k+1} = A u_k, given u_0 = (1, 0). As $k \to \infty$, does the solution "blow up" to infinity, decay to zero, or approach a non-zero neutrally stable state?

c) Solve the differential equation du/dt = Au for the same matrix A and the same initial condition $u(0)$ = (1, 0). As $t \to \infty$, does the solution "blow up" to infinity, decay to zero, or approach a non-zero neutrally stable state?

3. (14) Find the co-factor matrix for A=$\begin{bmatrix} 3 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$, and use it to calculate det A and A^{-1}.

Finally, use the Cramer's rule to solve Ax=$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

4. (18) Consider three data points in the functional dependence $b = y(t)$:

$y(-1) = 0$, $y(0) = 1$, $y(1) = -3$

a) Write down the over-constrained (and hence unsolvable) system Ax = b, where x=[C D]^T is the vector of unknown coefficients of the linear regression y = C+D t
b) Find the matrix projecting any vector onto the column space of A
c) Find the linear regression coefficients C and D. Plot the line and the data points.

5. (10) Choose **one** of the following two questions:

a) If AB+AC=D, find the expression for the inverse of A (assume all matrices are square and invertible)

b) Analyze the stability of the zero equilibrium of the system {dw/dt = u, du/dt=2u+w}. Is there an initial condition for which the solution decays to zero? Illustrate the dynamics in the phase plane.

6. (10) For each property listed below, indicate whether it is satisfied by any projection matrix (A=P) or any rotation matrix (A=Q), or neither. You do not have to construct any example matrices. If you are sure about the correctness of your answer, you don't have to explain.

a) $A^2=A$
b) $A^2=I$
c) $A^{-1}=A^T$
d) A is singular
e) $A=A^T$
f) A is non-diagonalizable
g) det A = 0
h) det A = 1
i) N(A) = { 0 }
j) C(A) = { 0 }