

Moth 630 Linear Algebra

Quiz # 2

Problem 1

① $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ singular

② $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ nonsingular

Problem 2

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

rank: 2

0 free variables

Solvability condition: $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \rightarrow \begin{bmatrix} b_1 \\ b_2 \\ b_3 - 2b_1 - 3b_2 \end{bmatrix}$

$$b_3 - 2b_1 - 3b_2 = 0$$

unique solution

Problem 3

$$\begin{bmatrix} 1 & 2 & 0 & 2 & 1 \\ -1 & -2 & 1 & 1 & 0 \\ 1 & 2 & -3 & -7 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 0 & 2 & 1 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & -3 & -9 & -3 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 2 & 0 & 2 & 1 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank: 2

Left Null: dimension 1

Two pivot variables three free variables

Column space & Row space: Dimension 2: $N(A)$: Dimension 3

