

Quiz #3 Moth 630 Linear Algebra

Problem 1

length $\|a\| = \sqrt{9} = 3$
 use Null space base vector calculation.

$$(2 \ -2 \ 1) \rightarrow (1 \ -1 \ \frac{1}{2})$$

$F = (-1 \ \frac{1}{2})$, two vectors

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{1}{2} \\ 0 \\ 1 \end{bmatrix} \checkmark$$

Problem 2

$$\underline{q}_1 = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

$$\underline{q}_2 = \frac{(a_2 - \underline{q}_1^T a_2 \underline{q}_1)}{\|a_2 - \underline{q}_1^T a_2 \underline{q}_1\|} = \begin{bmatrix} \sin\theta \\ -\cos\theta \end{bmatrix} \checkmark$$

$$R_{11} = \underline{q}_1^T a_1 = 1 \quad R_{21} = 0 \quad \checkmark$$

$$R_{21} = \underline{q}_1^T a_2 = \cos\theta \sin\theta, \quad R_{22} = \sin^2\theta$$

Problem 3

$$\underline{A} = \begin{bmatrix} 4 & 1 \\ 5 & 2 \\ 2 & 0 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} \frac{4}{7} & -\frac{1}{7} \\ \frac{5}{7} & \frac{4}{7} \\ \frac{2}{7} & -\frac{4}{7} \\ \frac{2}{7} & -\frac{4}{7} \end{bmatrix} \begin{bmatrix} 7 & 2 \\ 0 & 1 \end{bmatrix} \checkmark$$

Problem 4

$$(a) \underline{A} = \underline{Q}\underline{R} = \begin{bmatrix} \frac{1}{\sqrt{10}} & -\frac{3}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \\ \frac{4}{\sqrt{10}} & \frac{4}{\sqrt{10}} \\ \frac{5}{\sqrt{10}} & -\frac{5}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix} \begin{bmatrix} 10 & 10 \\ 0 & 10 \end{bmatrix} \checkmark$$

$$(b) \bar{x} = (\underline{A}^T \underline{A})^{-1} \underline{A}^T \underline{b} = \underline{R}^{-1} \underline{Q}^T \underline{b} = \begin{pmatrix} 10 & 10 \\ 0 & 10 \end{pmatrix}^{-1} \begin{pmatrix} 5 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix}$$

$$\underline{A}^T \underline{A} = \underline{R}^T \underline{Q}^T \underline{Q} \underline{R} = \underline{R}^T \underline{R} = \underline{R}^{-1} \underline{R}^T \underline{R}^T \underline{Q}^T$$

back substitution

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