

MATH 630 Linear Algebra

1/2

Quiz #6

Problem 1

$$A = \begin{bmatrix} 3 & 6 \\ 6 & 16 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$$

diag matches with the coefficients of the squares
 l_{ji} $i \leq j \leq 2$, $1 \leq i \leq 2$ matches with the combinations of the squares.

Problem 2

$$A = Q \Lambda Q^T, \quad A^2 = Q \Lambda^2 Q^T, \quad A^{-1} = Q \Lambda^{-1} Q^T$$

$$\lambda_i > 0 \Rightarrow \lambda_i^2, \quad 1/\lambda_i > 0$$

Problem 3

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 4 & 9 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & -2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} \underline{1} & 2 & 3 \\ 0 & \underline{1} & -2 \\ 0 & 0 & \underline{-4} \end{bmatrix}$$

A : indefinite, so is A^{-1}

$$B = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 6 & -2 & 0 \\ 0 & -2 & 5 & -2 \\ 0 & 0 & -2 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & -2 & 5 & -2 \\ 0 & 0 & -2 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & -2 & 3 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} \underline{1} & 2 & 0 & 0 \\ 0 & \underline{2} & -2 & 0 \\ 0 & 0 & \underline{3} & -2 \\ 0 & 0 & 0 & \underline{\frac{5}{3}} \end{bmatrix}$$

all pivots are positive $\Rightarrow B$ positive definite, so $C = -B$ is negative definite.

There is a real solution for $-x^T A x = 1$.

Problem 4

$$\underline{A} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & -1 & -1 \\ 0 & \frac{3}{2} & -\frac{3}{2} \\ 0 & -\frac{3}{2} & \frac{3}{2} \end{bmatrix} \rightarrow \begin{bmatrix} 2 & -1 & -1 \\ 0 & \frac{3}{2} & -\frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

$$\underline{L} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ -\frac{1}{2} & -1 & 1 \end{bmatrix}$$

$$\underline{x}^T \underline{A} \underline{x} = 2 \left(x_1 - \frac{1}{2} x_2 - \frac{1}{2} x_3 \right)^2 + \frac{3}{2} (x_2 - x_3)^2$$

$$\underline{B} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\underline{L} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\underline{x}^T \underline{B} \underline{x} = (x_1 + x_2 + x_3)^2$$