Math 630 - Linear Algebra and Its Applications

Instructor: Prof. X. Sheldon Wang
Quiz 2
(Closed book)

Assigned: 8:00pm, Feb. 17, 2005
Due: 9:00pm, Feb. 17, 2005

Problem 1 (25 points)

Show that the set of nonsingular 2×2 matrices is not a vector space. Show also that the set of singular 2×2 matrices is not a vector space.

Problem 2 (25 points)

Describe the set of attainable right sides **b** for
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix},$$
by finding the constraints on **b** that turn the third equation into $0 = 0$ (after elimination). What is the rank? How many free variables, and how many solutions?

Problem 3 (25 points)

What is the echelon form **U** of
$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 0 & 2 & 1 \\ -1 & -2 & 1 & 1 & 0 \\ 1 & 2 & -3 & -7 & -2 \end{bmatrix}?$$
What are the dimensions of its four fundamental subspaces?

Problem 4 (25 points)

(a) Find the rank of **A**, and give a basis for its nullspace.
$$\mathbf{A} = \mathbf{LU} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 \\ 3 & 2 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 & 1 & 2 & 1 \\ 0 & 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(b) True or False. The first 3 rows of **U** are a basis for the row space of **A**. True or False. Columns 1, 3, 6 of **U** are a basis for the column space of **A**. True or False. The four rows of **U** are a basis for the row space of **A**.

(c) Find as many linearly independent vectors **b** as possible for which $\mathbf{Ax} = \mathbf{b}$ has a solution.

(d) In elimination on **A**, what multiple of the third row is subtracted to knock out the fourth row?