Math 630 - Linear Algebra and Its Applications

Instructor: Prof. X. Sheldon Wang

Quiz 4

(Closed book)

Assigned: 8:00pm, Mar. 31st, 2005
Due: 9:00pm, Mar. 31st, 2005

Problem 1 (25 points)

Use row operations to calculate the determinant of the 3×3 Vandermonde matrix

$$det \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}.$$

Problem 2 (25 points)

Use the cofactor matrix to invert

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

Problem 3 (25 points)

If $\mathbf{B} = \mathbf{M}^{-1}\mathbf{A}\mathbf{M}$, why is $det\mathbf{B} = det\mathbf{A}$? Show also that $det\mathbf{A}^{-1}\mathbf{B} = 1$.

Problem 4 (25 points)

How are $det(2\mathbf{A})$, $det(-\mathbf{A})$, and $det(\mathbf{A}^2)$ related to $det(\mathbf{A})$, when $\mathbf{A}$ is $n \times n$?