Math 630 - Linear Algebra and Its Applications

Instructor: Prof. X. Sheldon Wang

Quiz 5

(Closed book)

Assigned: 8:00pm, April 14th, 2005
Due: 9:00pm, April 14th, 2005

Problem 1 (25 points)

Find the eigenvalues and eigenvectors of the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}.$$

Verify that the trace equals the sum of the eigenvalues, and the determinant equals their product. Suppose we shift the preceding matrix $\mathbf{A}$ by subtracting $7\mathbf{I}$:

$$\mathbf{B} = \mathbf{A} - 7\mathbf{I} = \begin{bmatrix} -6 & -1 \\ 2 & -3 \end{bmatrix}.$$

What are the eigenvalues and eigenvectors of $\mathbf{B}$, and how are they related to those of $\mathbf{A}$.

Problem 2 (25 points)

Factor the following matrix into $\mathbf{S\Lambda S}^{-1}$:

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}.$$

Find the matrix $\mathbf{B}$ whose eigenvalues are 1 and 4, and the corresponding eigenvectors are $(3 \quad 1)^T$ and $(2 \quad 1)^T$, respectively.

Problem 3 (25 points)

If each number is the average of the two pervious numbers, $G_{k+2} = (G_{k+1} + G_k)/2$, set up the matrix $\mathbf{A}$ and diagonalize it. Starting from $G_o = 0$ and $G_1 = 1/2$, find the formula for G_k and compute its limit as $k \to \infty$.

Problem 4 (25 points)

Which of these matrices cannot be diagonalized?

$$\mathbf{A}_1 = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} 2 & 0 \\ 2 & -2 \end{bmatrix}, \quad \mathbf{A}_3 = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}.$$

Diagonalize the 2×2 skew-Hermitian matrix $\mathbf{K}$, whose entries are all i. Compute $e^{\mathbf{K}t} = \mathbf{S}e^{\mathbf{\Lambda}t}\mathbf{S}^{-1}$, and verify that $e^{\mathbf{K}t}$ is unitary. What is its derivative at $t = 0$?