Math 630 - Linear Algebra and Its Applications

Instructor: Prof. X. Sheldon Wang

Quiz 6

(Closed book)

Assigned: 8:00pm, April 28th, 2005
Due: 9:00pm, April 28th, 2005

Problem 1 (25 points)

The quadratic $f = 3(x_1 + 2x_2)^2 + 4x_2^2$ is positive. Find its matrix $\mathbf{A}$, factor it into $\mathbf{LDL}^T$, and connect the entries in $\mathbf{D}$ and $\mathbf{L}$ to the original f.

Problem 2 (25 points)

Show from the eigenvalues that if $\mathbf{A}$ is positive definite, so are $\mathbf{A}^2$ and $\mathbf{A}^{-1}$.

Problem 3 (25 points)

Decide whether the following matrices are positive definite, negative definite, semi-definite, or indefinite:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 4 & 9 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 6 & -2 & 0 \\ 0 & -2 & 5 & -2 \\ 0 & 0 & -2 & 3 \end{bmatrix}, \quad \mathbf{C} = -\mathbf{B}, \quad \mathbf{D} = \mathbf{A}^{-1}.$$

Is there a real solution to $-x^2 - 5y^2 - 9z^2 - 4xy - 6xz - 8yz = 1$?

Problem 4 (25 points)

For the semi-definite matrices

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \text{(rank 2)} \quad \text{and } \mathbf{B} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \text{(rank 1)}$$

write $\mathbf{x}^T\mathbf{A}\mathbf{x}$ as a sum of two squares and $\mathbf{x}^T\mathbf{B}\mathbf{x}$ as one square.