

Math 712, **Homework Set 8**, November 7, 2005
Due Wednesday, November 16

1. Do Problems 7.1.4, 7.2.2, and 7.2.5 from the textbook.
2. Consider Fisher's equation $u_t = u_{xx} + u(1 - u)$; we know this nonlinear diffusion equation has travelling wave solutions (the steady state $u = 0$ is unstable to small perturbations while the $u = 1$ steady state is stable). Discretize the linear part of Fisher's equation with the Crank-Nicolson method; treat the nonlinear part explicitly, i.e., $u(1 - u) \sim U_m^n(1 - U_m^n)$. Use the Thomas algorithm in a code that solves the initial value problem over $x \in (-10, 10)$ with homogeneous Dirichlet boundary conditions at the end points; for an initial condition choose a positive smooth-bump function that is zero in $|x| > 0.5$. Is there a stability condition? Note: the preceding question does not have an obvious answer. Experiment with the discretization parameters in order to get traveling waves of constant speed moving to the left/right of the origin. Produce a graph of the solution versus x computed at enough values of t to demonstrate this travelling wave. Is the travelling wave what you expect from theory?