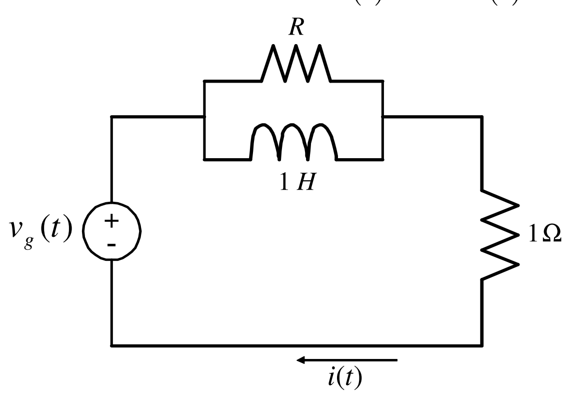
ECE 232 - Circuits and Systems II
Final

Please provide clear and complete answers. Don't forget to specify the units of measure!

1. (2 points) Using the Bode plot, write a transfer function $H(s)$ with the following properties: *i*) The amplitude response $|H(j\omega)|$ is (approximately) equal to $0dB$ for all frequencies between 1 and 10 rad/s; *ii*) The amplitude response $|H(j\omega)|$ is (approximately) smaller than $-20dB$ for all frequencies smaller than 0.1 and for all frequencies larger than 100 rad/s.

2. (2 points) What is the convolution between $x(t) = e^{-t}u(t)$ and $h(t) = \delta(t)$? What is the convolution between $x(t) = e^{-t}u(t)$ and $h(t) = 2\delta(t-1)$?



3. (5 points) Consider the circuit in the figure.
a. Evaluate the transfer function $H(s)$ between input $v_g(t)$ and output $i(t)$ as a function of the resistance R.
b. Evaluate poles and zeros of $H(s)$ as a function of R. Write and plot the Bode approximation of the amplitude response $H(j\omega)$ (as a function of R).
c. How would you choose R in order to make $H(s)$ a low-pass filter? What would be the 3-dB cut-off frequency?
d. Assuming $R = 1\Omega$ and $v_g(t) = 4\sin(2t)\ V$, evaluate $i(t)$ (for all $t \geq 0$, not only in the steady-state regime) using partial fraction expansion.
e. Assuming $R = 1\Omega$ and $v_g(t) = 3u(t)$, evaluate $i(t)$ in steady state.

4. (2 points) We want to design a band-pass filter based on a series RLC so that the quality factor is $Q = 4$, the central frequency is $20kHz$ and $C = 4\mu F$.
a. Draw a diagram for the circuit.
b. Find R and L.
c. Find the transfer function of the filter, calculate poles and zeros, and cut-off frequencies (check that the bandwidth is as required).
d. Sketch the frequency response.

Sol.:
1. We have

$$H(s) = \frac{s}{(1+s)(1+s/10)},$$

as follows from the usual rules for drawing Bode plots.

2. We have

$$e^{-t}u(t) * \delta(t) = e^{-t}u(t),$$

and

$$e^{-t}u(t) * 2\delta(t-1) = 2e^{-(t-1)}u(t-1).$$

3.

a. We have

$$\begin{aligned} H(s) &= \frac{1}{1+R||s} \\ &= \frac{1}{1+\frac{Rs}{R+s}} = \frac{s+R}{R+s(1+R)} \\ &= \frac{1}{1+R}\frac{s+R}{s+R/(1+R)}. \end{aligned}$$

b. We have zero $s=-R$ and pole $s=-R/(1+R)$. Moreover, we can write

$$H(j\omega) = \frac{j\omega/R+1}{j\omega(1+R)/R+1}$$

The Bode approximation is as follows:

- For $\omega \leq R/(1+R)$,
$$|H(j\omega)| \simeq 0dB;$$
- For $R/(1+R) \leq \omega \leq R$,
$$|H(j\omega)| \simeq -20\log(\omega(1+R)/R);$$
- For $\omega \geq R$,
$$|H(j\omega)| \simeq -20\log(1+R).$$

The plot can be done as usual.

c. We should choose $R \to \infty$ as it can be seen from the Bode plot. The transfer function is

$$H(s) = \frac{1}{s+1}$$

and the 3-dB cut-off frequency is 1 rad/s.

d. The transfer function is

$$H(s) = \frac{1}{2}\frac{s+1}{s+1/2},$$

and thus we can write

$$\begin{aligned} I(s) &= \frac{1}{2}\frac{s+1}{s+1/2}\frac{8}{s^2+4} \\ &= \frac{K_1}{s+1/2} + \frac{K_2}{s-j2} + \frac{K_2^*}{s+j2}. \end{aligned}$$

We calculate the constants as

$$K_1 = \frac{1}{2}(-1/2+1)\frac{8}{1/4+4} = \frac{8}{17}$$

and

$$\begin{aligned} K_2 &= \frac{1}{2}\frac{j2+1}{j2+1/2}\frac{8}{j2+j2} \\ &= 1.08\angle-102.5^\circ. \end{aligned}$$

The output is thus

$$\begin{aligned} i(t) &= \frac{8}{17}e^{-1/2t}u(t) + 2.17\cos(2t-90^\circ-102.52^\circ)u(t) \\ &= \frac{8}{17}e^{-1/2t}u(t) + 2.17\cos(2t+12.52^\circ)u(t). \end{aligned}$$

e. We have

$$i(t) = 3H(0)u(t) = 3u(t).$$

4. a. Please see textbook.
b. We can write

$$\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi \cdot 20 \cdot 10^3,$$

and thus

$$\begin{aligned} L &= \frac{1}{\omega_0^2 C} = \frac{1}{(2\pi)^2 \cdot 400 \cdot 10^6 \cdot 4 \cdot 10^{-6}} \\ &= \frac{1}{(2\pi)^2 \cdot 1600} = 1.58 \times 10^{-5} \\ &= 15.8\ \mu H. \end{aligned}$$

Moreover, we have

$$\begin{aligned} Q &= \frac{\omega_0}{\beta} = \frac{\omega_0}{2\alpha} \\ &= \frac{\omega_0}{2\alpha} = \frac{\omega_0}{2R/L} = 4, \end{aligned}$$

so that

$$\begin{aligned} R &= \frac{\omega_0}{8/L} = \frac{2\pi \cdot 20 \cdot 10^3}{8} \cdot 1.58 \cdot 10^{-5} \\ &= 0.25\ \Omega. \end{aligned}$$

c. As seen in class, we have

$$H(s) = \frac{2\alpha s}{s^2 + 2\alpha s + \omega_0^2}.$$

The rest of the calculation follows by using the usual formulas.