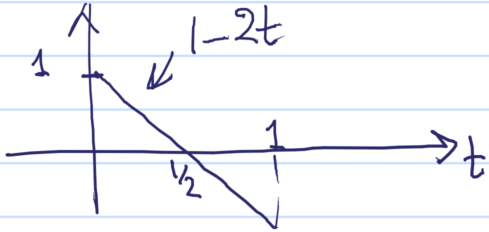
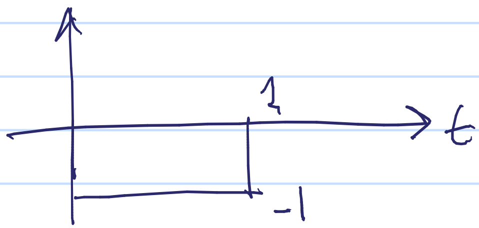
1. $x_{z,0}(t)$:



$x_{z,1}(t)$:

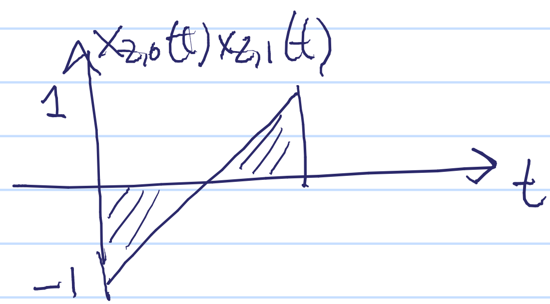


a. $E_0 = \int_0^1 (1-2t)^2 dt = \int_0^1 (1+4t^2-4t)\,dt = 1 + \frac{4t^3}{3} - 4\frac{t^2}{2}\Big|_0^1$

$= 1 + \frac{4}{3} - 2 = \frac{1}{3}$

$E_1 = 1$

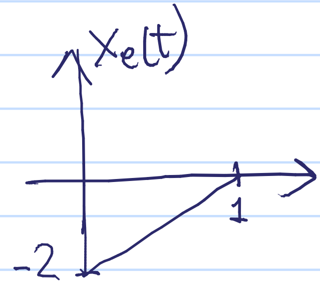
b. We have $\int_0^1 x_{z,0}(t)\, x_{z,1}(t)\, dt = 0$



$\Rightarrow \rho_{10} = 0$

$\Delta_E(1,0) = E_0 + E_1 = 1 + \frac{1}{3} = \frac{4}{3}$

c. Effective signal: $x_e(t) = x_{z,1}(t) - x_{z,0}(t)$

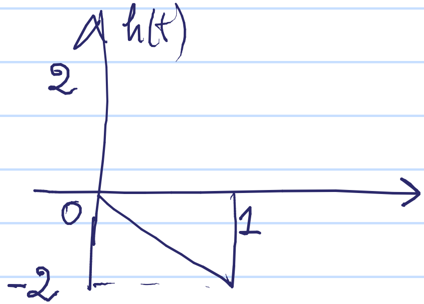


$$x_e(t) = -1-(1-2t) = -2+2t = -2(1-t)$$

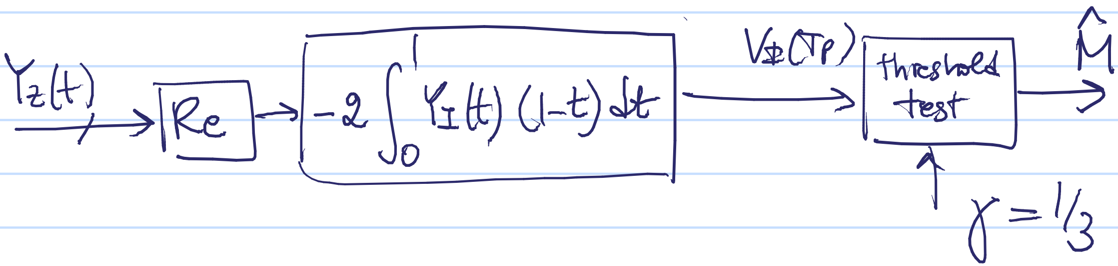
Matched filter

$$h(t) = x_e(-t+1)$$

$$= -2(\cancel{1}+t-\cancel{1}) = -2t$$



d. Using the correlator implementation



$$\gamma = \frac{E_1 - E_0}{2} = \frac{1-1/3}{2} = 1/3$$

e. Using the baseband demodulator at the previous point

$$V_I(T_p) = -2\int_0^1 (1-t)\,dt = -2\left(t-\frac{t^2}{2}\right)\Big|_0^1 = -2\left(\frac{1}{2}\right) = -1 < \frac{1}{3} \Rightarrow \hat{M}=0$$

f.

$$m_0 = \int_0^1 x_{I,0}(t)\,x_e(t)\,dt = -2\int_0^1 (1-2t)(1-t)\,dt$$

$$= -2\int_0^1 (1-3t+2t^2)\,dt = -2\left(t-\frac{3t^2}{2}+\frac{2t^3}{3}\Big|_0^1\right)$$

$$= -2\left(1-\frac{3}{2}+\frac{2}{3}\right) = -\frac{1}{3} = -E_0$$

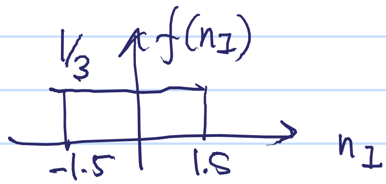
$$m_1 = -2\int_0^1 (-1)(1-t)\,dt = 2\int_0^1 (1-t)\,dt$$

$$= 2\left(t-\frac{t^2}{2}\right)\Big|_0^1 = 1 = E_1$$

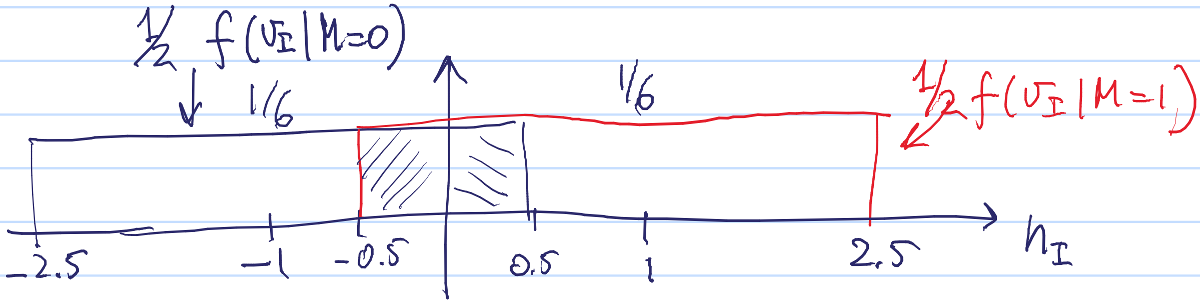
You can directly obtain $m_0 = -E_0$ and $m_1 = E_1$ from the formulas seen in class.

$$\sigma_{N_I}^2 = \frac{N_0}{2}\Delta_E(0,1) = \frac{1}{2}\,\frac{4}{3} = \frac{2}{3}$$

2. $V_I(T_P) = m_i + N_I$ with



a.

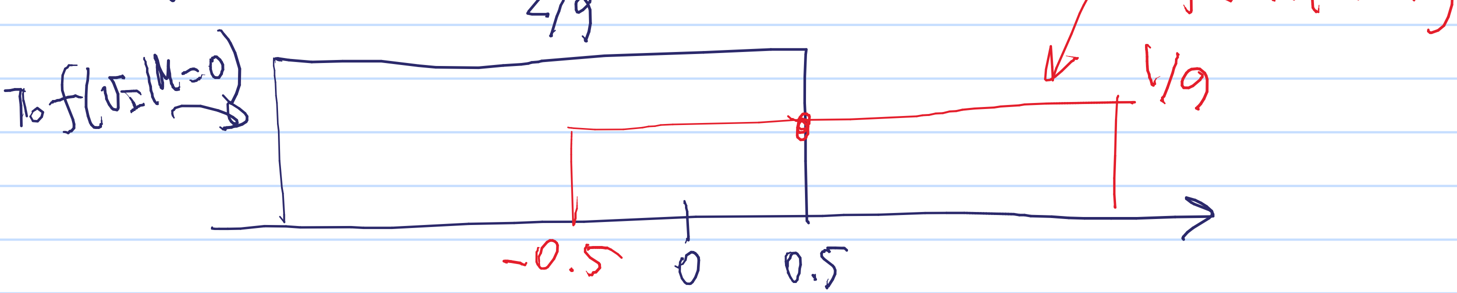


Following the same arguments seen in class, any threshold $-0.5 \le \gamma \le 0.5$ is optimal $\Rightarrow$ here we choose $\gamma = 0$

b. $$P_B(E) = \frac{1}{2} \Pr\left[V_I(T_P) \le 0 \mid M=1\right] + \frac{1}{2} \Pr\left[V_I(T_P) \ge 0 \mid M=0\right]$$

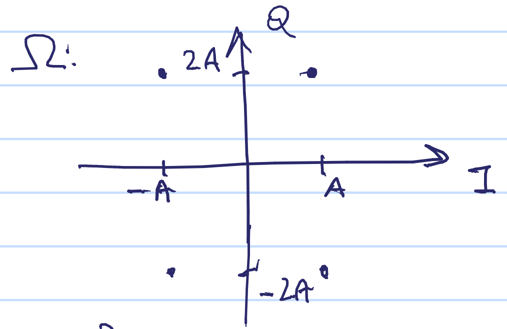
$$= \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$$

c. If $\pi_1 = 1/3$:



optimal threshold $\gamma = 0.5$

3. Ω:



a. $\frac{1}{4}\sum_{i=0}^{3} |d_i|^2 = 2 \Rightarrow (A^2 + 4A^2) = 2$

$\Rightarrow A^2 = 2/5 \rightarrow A = \sqrt{2/5}$

b. Conditional distance spectra:

for all points $\{(4A^2 E_b, 1), (16A^2 E_b, 1), (20A^2 E_b, 1)\}$

$\Leftrightarrow \{(\frac{8}{5}E_b, 1), (\frac{32}{5}E_b, 1), (8E_b, 1)\}$

Distance spectrum:

$$\{(\frac{8}{5}E_b, 4), (\frac{32}{5}E_b, 4), (8E_b, 4)\}$$

Union bound:

$$P_{WUB}(E) = \frac{1}{4}\left(2\,\mathrm{erfc}\left(\sqrt{\frac{2E_b}{5N_0}}\right) + 2\,\mathrm{erfc}\left(\sqrt{\frac{8E_b}{5N_0}}\right) + 2\,\mathrm{erfc}\left(\sqrt{\frac{2E_b}{N_0}}\right)\right)$$

$$\simeq \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{2E_b}{5N_0}}\right) = Q\left(\sqrt{\frac{4E_b}{5N_0}}\right)$$

c. Comparison with 4PSK $\left(P_{WUB}(E) \simeq \mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)\right)$

$$\text{loss} = 10\log_{10}\left(\underbrace{\frac{5}{2}}_{2.5}\right) \simeq 4\,\text{dB}$$