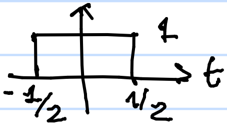
1. $Y(f) = \mathcal{F}\{\underbrace{x(t+1/2)}\,\sin(500\pi t)\}$



$$= \frac{1}{2}\,\mathrm{sinc}(f-250)\,e^{-j\pi/2} + \frac{1}{2}\,\mathrm{sinc}(f+250)\,e^{j\pi/2}$$

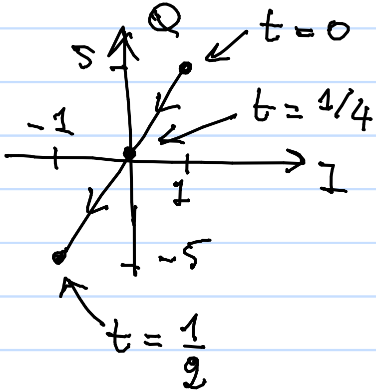
$$= \frac{1}{2j}\,\mathrm{sinc}(f-250) - \frac{1}{2j}\,\mathrm{sinc}(f+250)$$

$$\arg(Y(250)) = -\frac{\pi}{2}$$

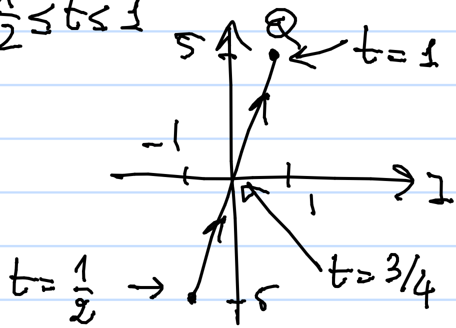
2. $x_1(t) = \cos(2\pi t)$

$x_2(t) = 5\cos(2\pi t)$

a. for $0 \le t \le \frac{1}{2}$



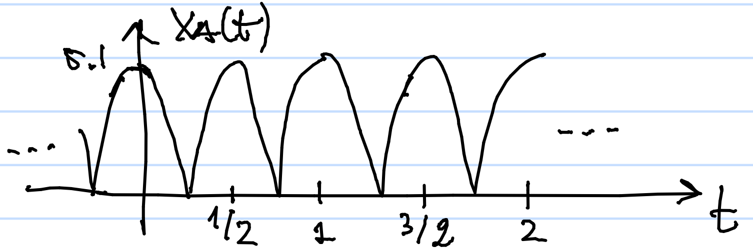
for $\frac{1}{2} \le t \le 1$

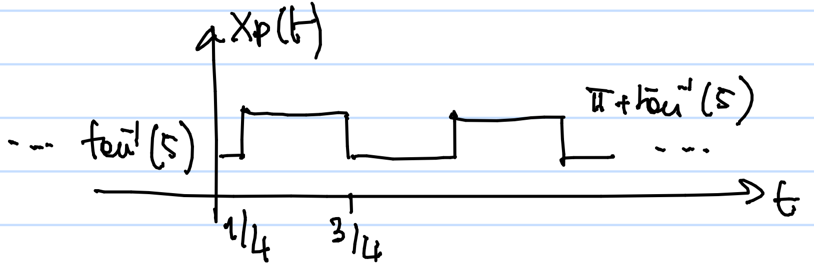


and then it repeats periodically.

b. $$X_A(t) = \sqrt{(\cos(2\pi t))^2 + 25\,(\cos(2\pi t))^2}$$

$$= \sqrt{26}\,|\cos(2\pi t)| \approx 5.1\,|\cos(2\pi t)|$$





$\tan^{-1}(5) = 78.7° = 1.37$ rad

3. a. $\left(\sqrt{2}\, x_I(t)\cos(20\pi t) - \sqrt{2}\, x_Q(t)\sin(20\pi t)\right) \times \cos(20\pi t + \pi/4)$

$$= \sqrt{2}\, x_I(t)\left(\frac{1}{2}\cos\left(\frac{\pi}{4}\right)\right) - \sqrt{2}\, x_Q(t)\left(\frac{1}{2}\sin\left(-\frac{\pi}{4}\right)\right)$$

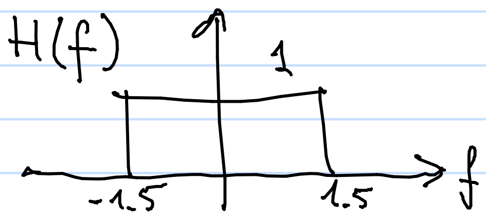
$+$ higher-order terms

$\Rightarrow$ after filtering, we obtain

$$\frac{1}{\sqrt{2}} x_I(t)\cos\left(\frac{\pi}{4}\right) + \frac{1}{\sqrt{2}} x_Q(t)\sin\left(\frac{\pi}{4}\right)$$

$$= \frac{1}{2} x_I(t) + \frac{1}{2} x_Q(t)$$

It cannot serve directly as a downconverter since the output is a mixture of both $x_I(t)$ and $x_Q(t)$.

b. 

$\Rightarrow h(t) = 3\,\mathrm{sinc}(3t)$

4. $\frac{1}{T_s} \geq 2\,(\text{largest frequency}) = 2 \times 11\ \text{Hz}$

$\Rightarrow T_s \leq \frac{1}{22}\ \text{s}$

For how to plot the Fourier transform, see lecture slides.