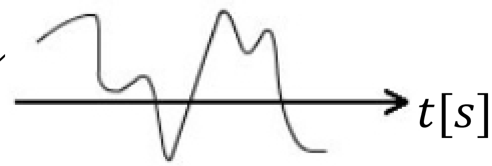
NJIT

New Jersey's Science & Technology University

THE EDGE IN KNOWLEDGE

Review of Signals & Systems

time domain

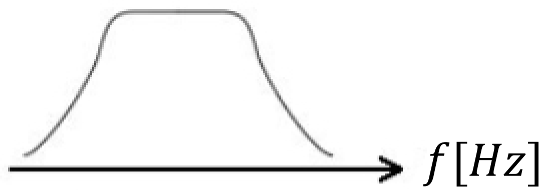


oscilloscope

Fourier transform

Signal analysis

frequency domain



spectrum analyzer

Ex.: frequency-domain view of radio signals
(http://njit2.mrooms.net/mod/page/view.php?id=32810)

Signal Classification

For first part of this course

- $\boxed{\text{Deterministic}}$ vs. random
- Energy vs. Power
- Periodic vs. aperiodic
- Complex vs. real
- Continuous-time vs. discrete-time

Energy vs. Power Signals

- Energy of a signal $x(t)$

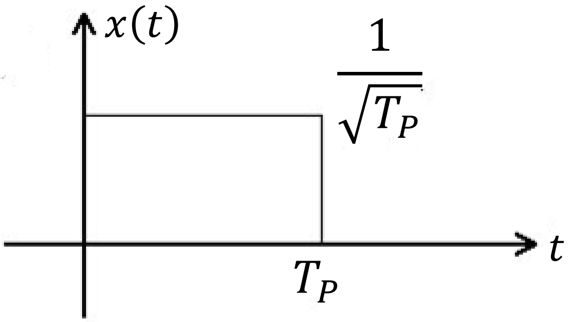
$$E_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt$$

- $x(t)$ is measured in Amperes (A) or Volts (V)
  - $\rightarrow$ E_x is measured in A^2s or V^2s
  - $\rightarrow$ Assuming that the load has unit resistance, we will measure E_x in Joules (J)

Examples

a) $$x(t) = \begin{cases} \frac{1}{\sqrt{T_p}}, & 0 \le t \le T_p \\ 0, & elsewhere \end{cases}$$

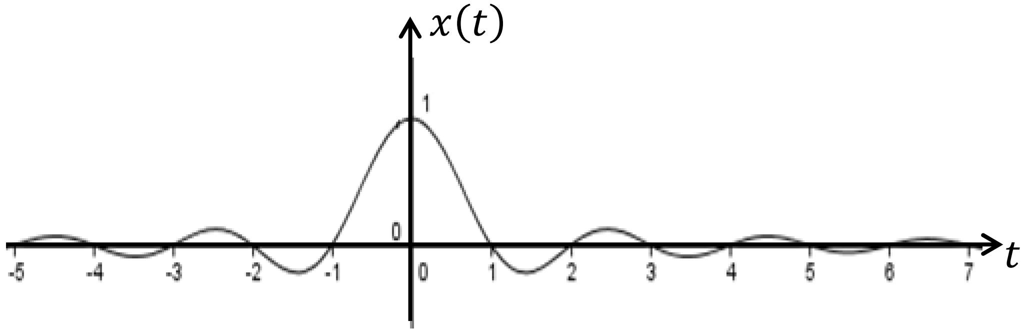
$$E_x = 1\,J$$



rectangular waveform

b) $$x(t) = sinc(t) = \frac{\sin(\pi t)}{\pi t}$$

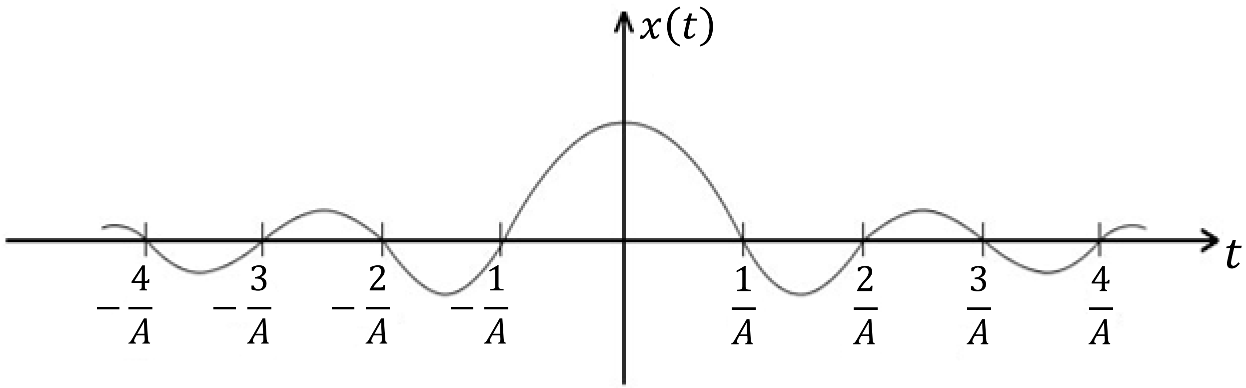
$$E_x = 1\,J$$



sinc waveform

Examples

c) $x(t) = sinc(At) = \dfrac{\sin(\pi At)}{\pi At}$ for $A > 0$ $\qquad E_x = \dfrac{1}{A}$



for $A > 1$: "shrinking" $\longrightarrow$ less energy
for $A < 1$: "stretching" $\longrightarrow$ more energy

d) Voice signal in Figure 2.1

Energy vs. Power Signals

- Any signal with $E_x < +\infty$ i.e., with finite energy, is called an **energy signal**

Energy vs. Power Signals

- The sine and cosine functions are examples of power signals

- Power of a signal $x(t)$

$$P_x = \lim_{T_m \to \infty} \frac{1}{T_m} \int_{-\frac{T_m}{2}}^{\frac{T_m}{2}} |x(t)|^2 dt$$

- P_x measured in Watts assuming unit resistance

Energy vs. Power Signals

- A signal with $0 < P_x < +\infty$, i.e., finite non-zero power, is a **power signal**

- Note that:
  - An energy signal is **not** a power signal:

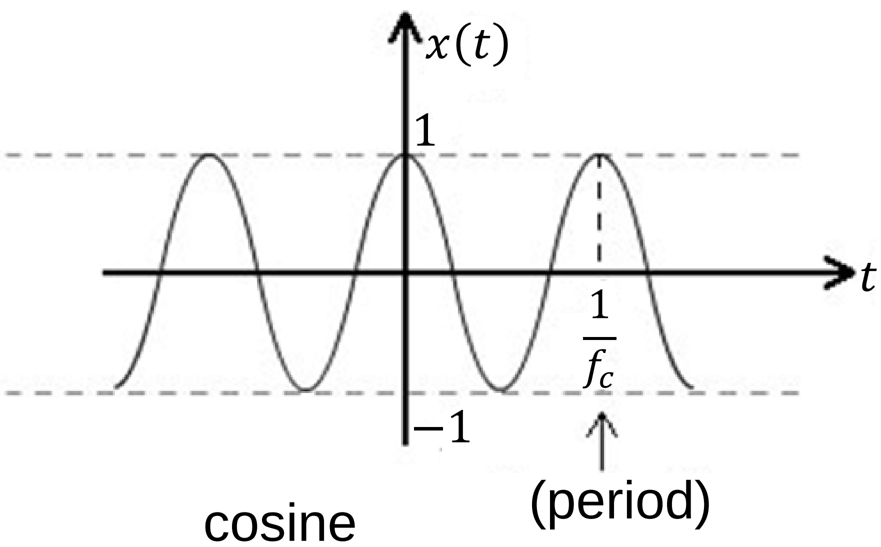
$$E_x < +\infty \longrightarrow P_x = 0$$

  - A power signal is **not** an energy signal:

$$P_x > 0 \longrightarrow E_x = +\infty$$

Examples

a) $x(t) = \cos(2\pi f_c t)\ ,\ -\infty < t < +\infty$

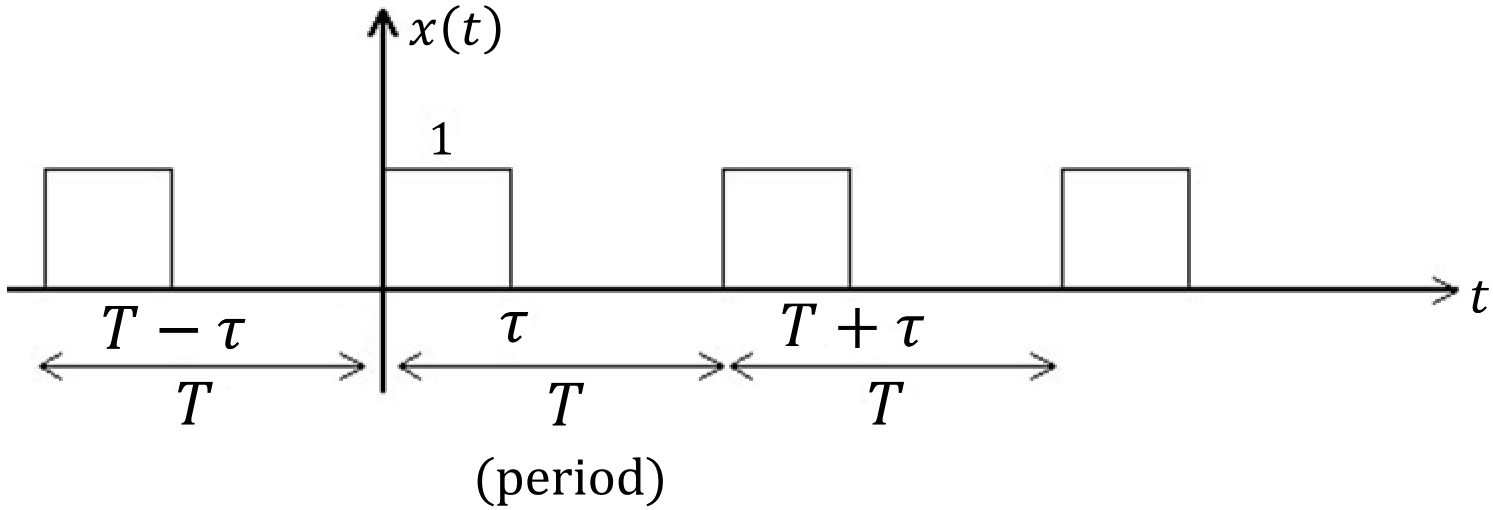


$$P_x = \frac{1}{2} W$$

(See example 2.4)

Examples

b) Periodic pulse train



$P_x = \frac{\tau}{T}$ (See example 2.5)

Periodic vs. Aperiodic Signals

- $x(t)$ is periodic if

$$x(t) = x(t + nT_0) \quad \text{for some } T_0 \neq 0 \text{ and all integers } n \qquad (*)$$

- Period:

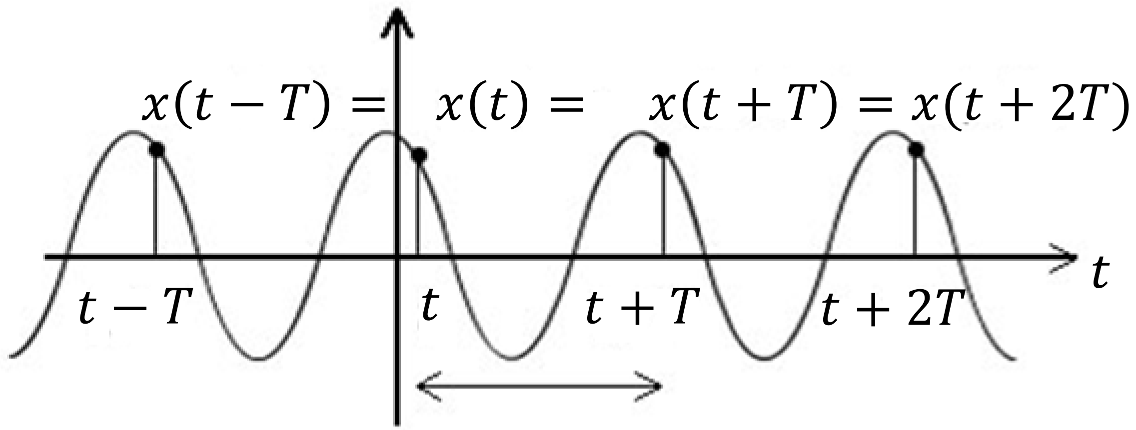
$$T = min\{\ |T_0| \text{ such that } (*) \text{ holds } \ \}$$

- Fundamental frequency:

$$f_T = \frac{1}{T}$$

Example

$$x(t) = \cos(2\pi f_c t)\,, -\infty < t < t\infty$$



$$T = \frac{1}{f_c} \text{ period}$$

$$T_0 = \frac{n}{f_c} \text{ with n integer}$$

$$f_c = \text{fundamental frequency}$$

Periodic vs. Aperiodic Signals

- For any periodic signal:

$$P_x = \frac{E_{x(t),0 \le t \le T}}{T}$$

energy of the signal in one period

Real vs. Complex Signals

- All previous examples are real signals

- Complex signal

$$z(t) = x(t) + j\,y(t)$$

cartesian representation

real part
Re $[z(t)]$

imaginary part
Im $[z(t)]$

$\longrightarrow$ $x(t)$ and $y(t)$ are real signals

Real vs. Complex Signals

polar representation

$$z(t) = \alpha(t)e^{j\theta(t)}$$

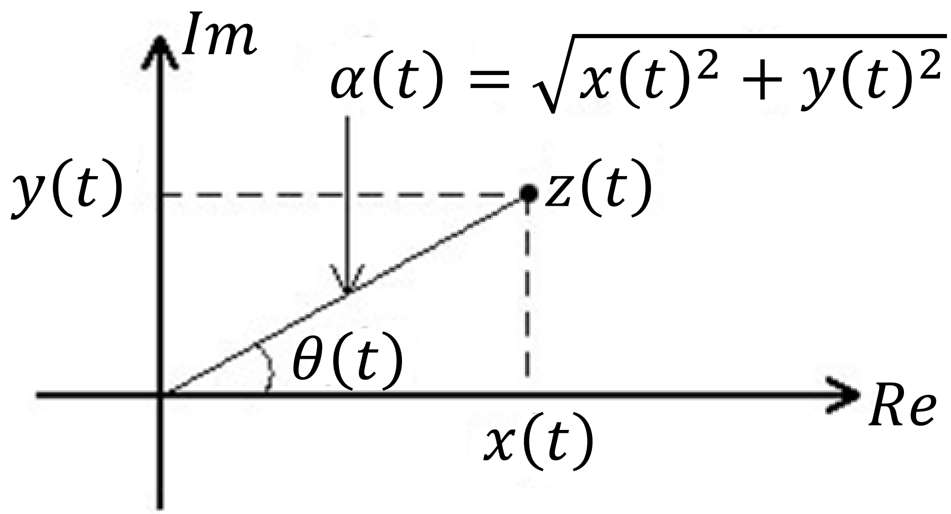
magnitude
or amplitude
$|z(t)|$

phase
$\arg(z(t))$

$\longrightarrow$ $\alpha(t)$ and $\theta(t)$ are real signals

Real vs. Complex Signals

- Relationship between the two representations



Example

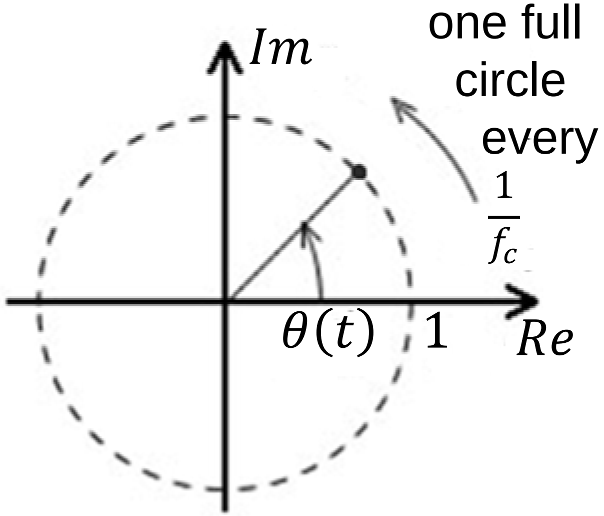
$$z(t) = e^{j2\pi f_c t} = \cos(2\pi f_c t) + jsin(2\pi f_c t)$$

$$Re[z(t)] = \cos(2\pi f_c t)$$

$$Im[z(t)] = \sin(2\pi f_c t)$$

$$|z(t)| = 1$$

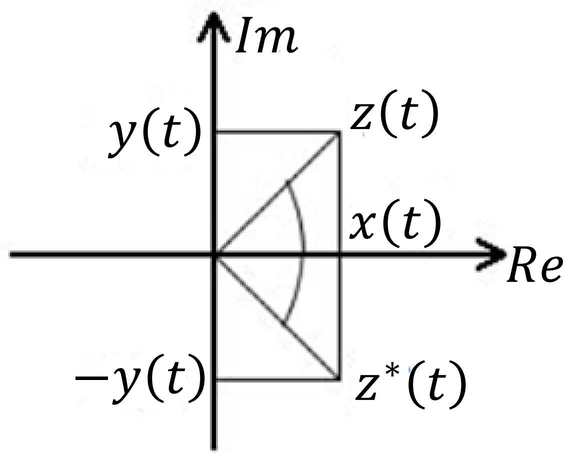
$$\theta(t) = 2\pi f_c t$$



Real vs. Complex Signals

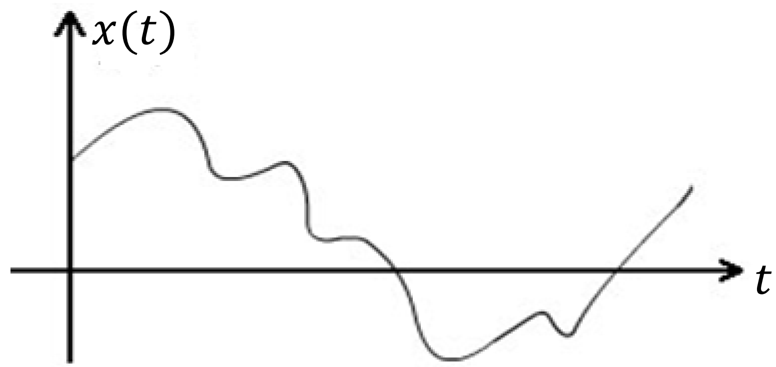
- Complex conjugate

$$z^*(t) = x(t) - j\,y(t) = \alpha\,(t)e^{-j\theta(t)}$$

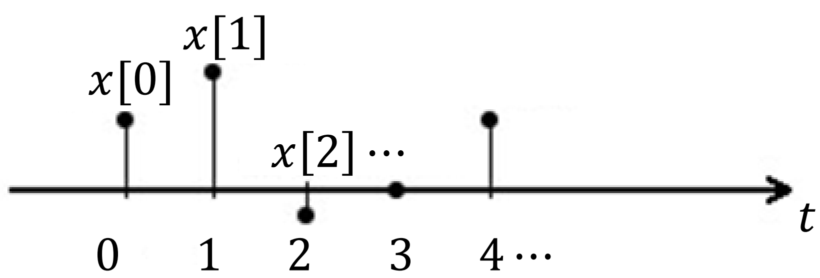


- See equations (2.17) in textbook for properties

Continuous vs. Discrete-time Signals



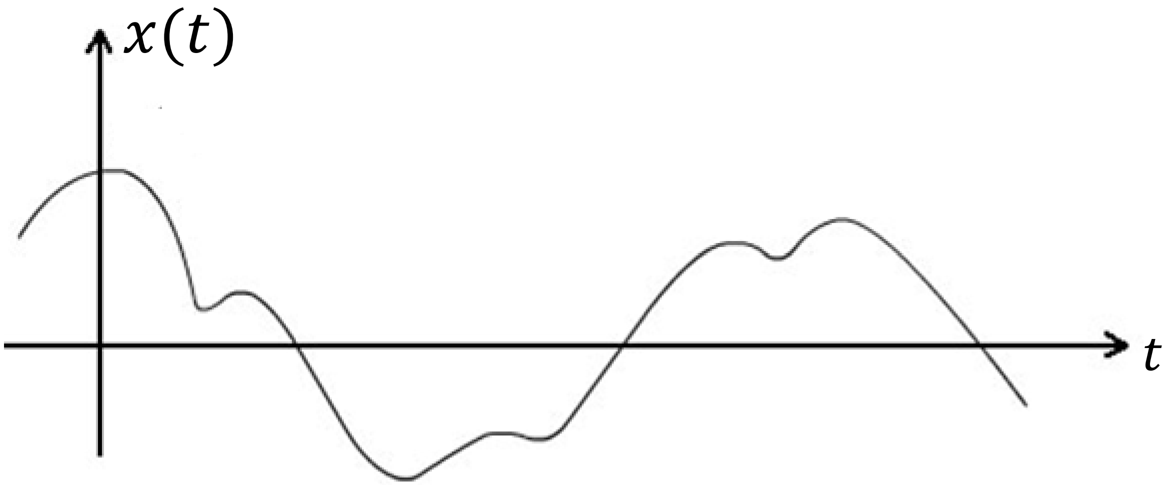
continuous-time $x(t)$
ex: radio, light,…



discrete-time $x[k]$
ex: sampled signal

Continuous vs. Discrete-time Signals

- A discrete-time signal is often obtained by sampling a continuous-time signal



continuous-time signal $x(t)$

Continuous vs. Discrete-time Signals

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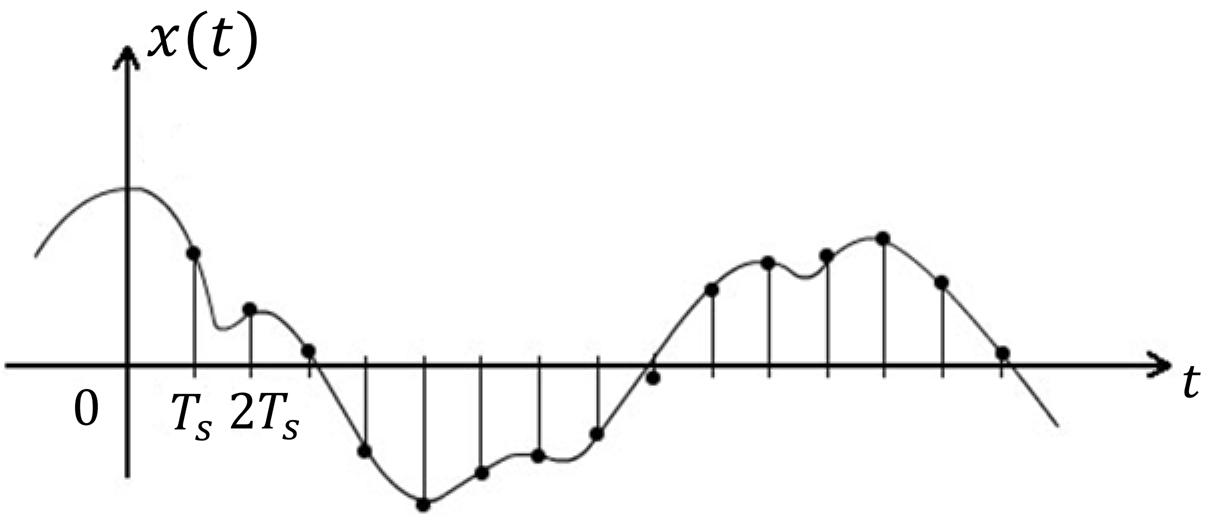

Sampling T_s = sampling period

Continuous vs. Discrete-time Signals

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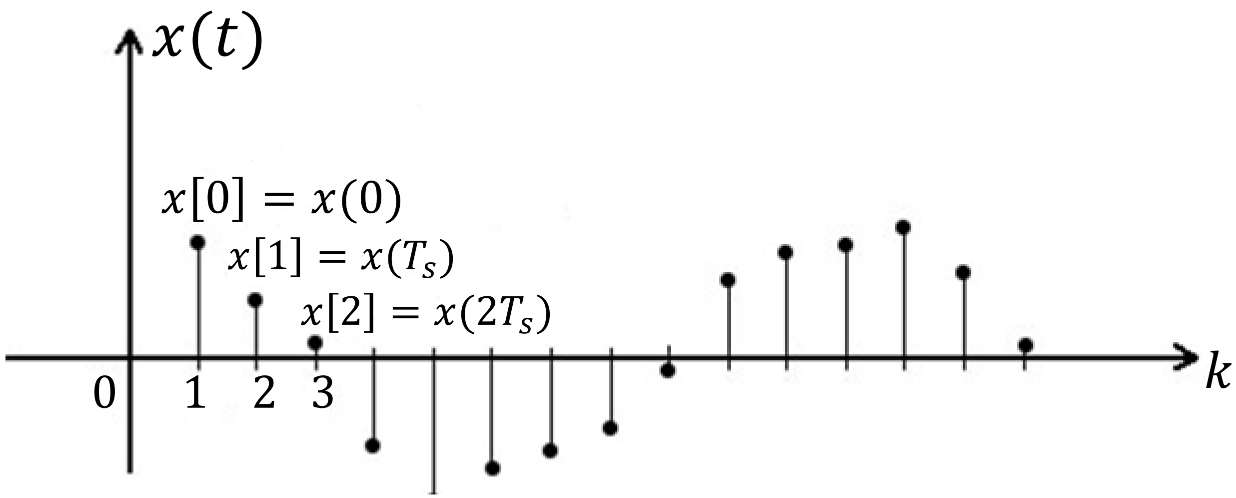

discrete-time signal $x[k]$

- Sampling is necessary in order to allow for digital signal processing, e.g., via MATLAB

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