NJIT

New Jersey's Science & Technology University

THE EDGE IN KNOWLEDGE

- When K_b is large, calculating the union bound may be complicated. Moreover, the bound may be meaningless (i.e., larger than 1). In this case, it is often useful to calculate the union bound approximation.
- Union bound approximation:

  idea – the most likely mistakes happen between the closest symbols

- Minimum distance

$$\Delta_E(\min) = \min_{\substack{i,j: \\ i \neq j}} \Delta_E(i,j)$$

- Union bound approximation:

$$P_W(E) \simeq \frac{1}{2^{K_b}} \underbrace{A_d}_{\substack{\text{number} \\ \text{of message pairs} \\ \text{at the minimum distance}}}(\min)\, Q\left(\sqrt{\frac{\Delta_E(\min)}{2N_0}}\right)$$

Ex.: 4-PWM

Union bound approximation

$$P_W(E) \simeq \frac{3}{2}\,\mathrm{Q}\left(\sqrt{\frac{2E_b}{5N_0}}\right)$$

Linear modulation

- We have seen that, in general, we need 2^{K_b} correlators to implement MLWD.
- However, for M-PAM, only one correlator was sufficient.
- We will now see that all linear modulations have this property.

- Linear modulation:

$$x_{z,i}(t) = d_i \sqrt{E_b} u(t) \qquad i = 0,1,\dots,2^{K_b}-1$$

complex number
$d_i \in \Omega_d$:

$$\frac{1}{2^{K_b}} \sum_{i=0}^{2^{K_b-1}} |d_i|^2 = K_b$$

waveform of unit energy and of duration T_P

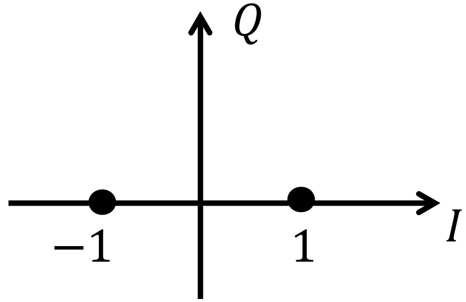
Ex.: $u(t) = \begin{cases} \frac{1}{\sqrt{T_P}} & 0 \le t \le T_P \\ 0 & elsewhere \end{cases}$

- Ω_d is called the constellation

Ex.: a) BPSK $(K_b = 1)$

$\Rightarrow$ the constellation is $\Omega_d = \{-1,1\}$

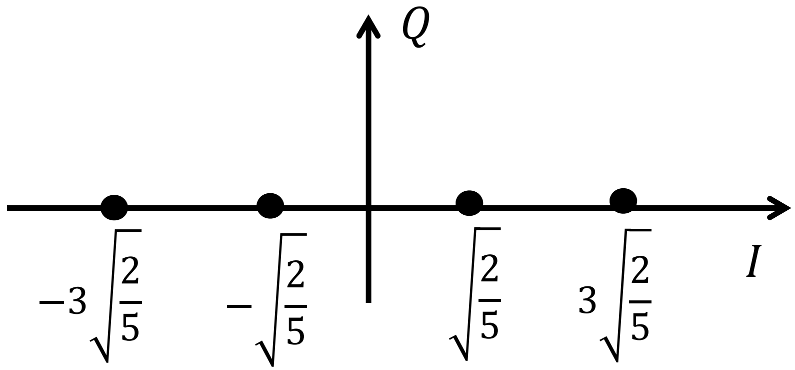
$$x_{z,i}(t) = \begin{cases} \pm\sqrt{\dfrac{E_b}{T_P}} & 0 \leq t \leq T_P \\ 0 & elsewhere \end{cases}$$



(Note that: $\frac{1}{2}\,(1^2 + 1^2) = K_b = 1$)

b) 4-PAM $(K_b = 2)$

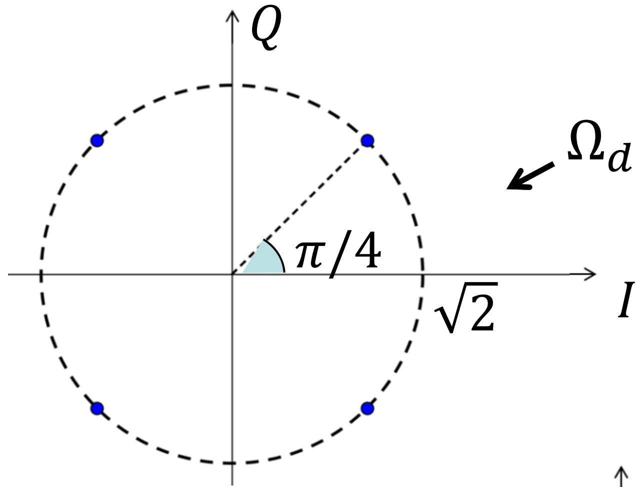
The constellation is $\Omega_d = \left\{\pm\sqrt{\frac{2}{5}}, \pm 3\sqrt{\frac{2}{5}}\right\}$



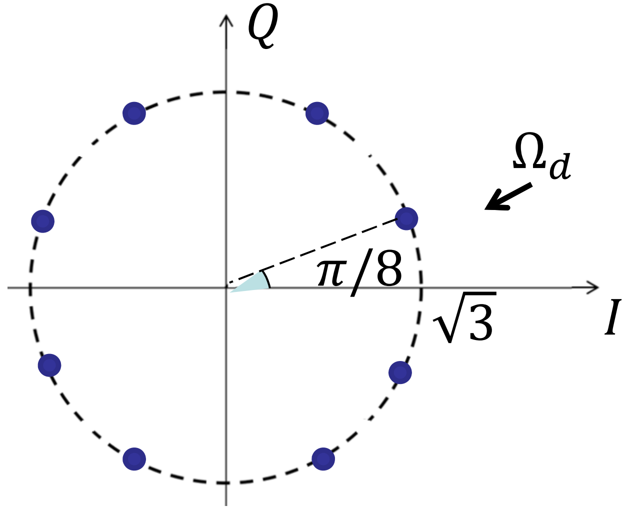
(Note that: $\frac{1}{4}\left(2 \cdot \frac{2}{5} + 2 \cdot \frac{18}{5}\right) = K_b = 2$)

c) 4-PSK $(K_b = 2)$

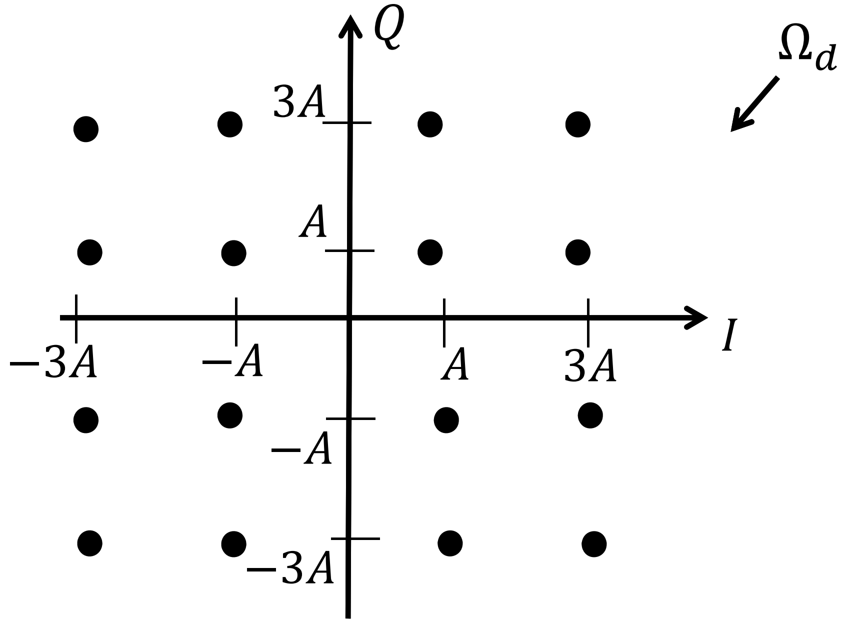
$$\Omega_d = \left\{\sqrt{2}e^{j\frac{\pi}{4}}, \sqrt{2}e^{j\frac{3\pi}{4}}, \sqrt{2}e^{-j\frac{\pi}{4}}, \sqrt{2}e^{-j\frac{3\pi}{4}}\right\}$$



d) 8-PSK $(K_b = 3)$



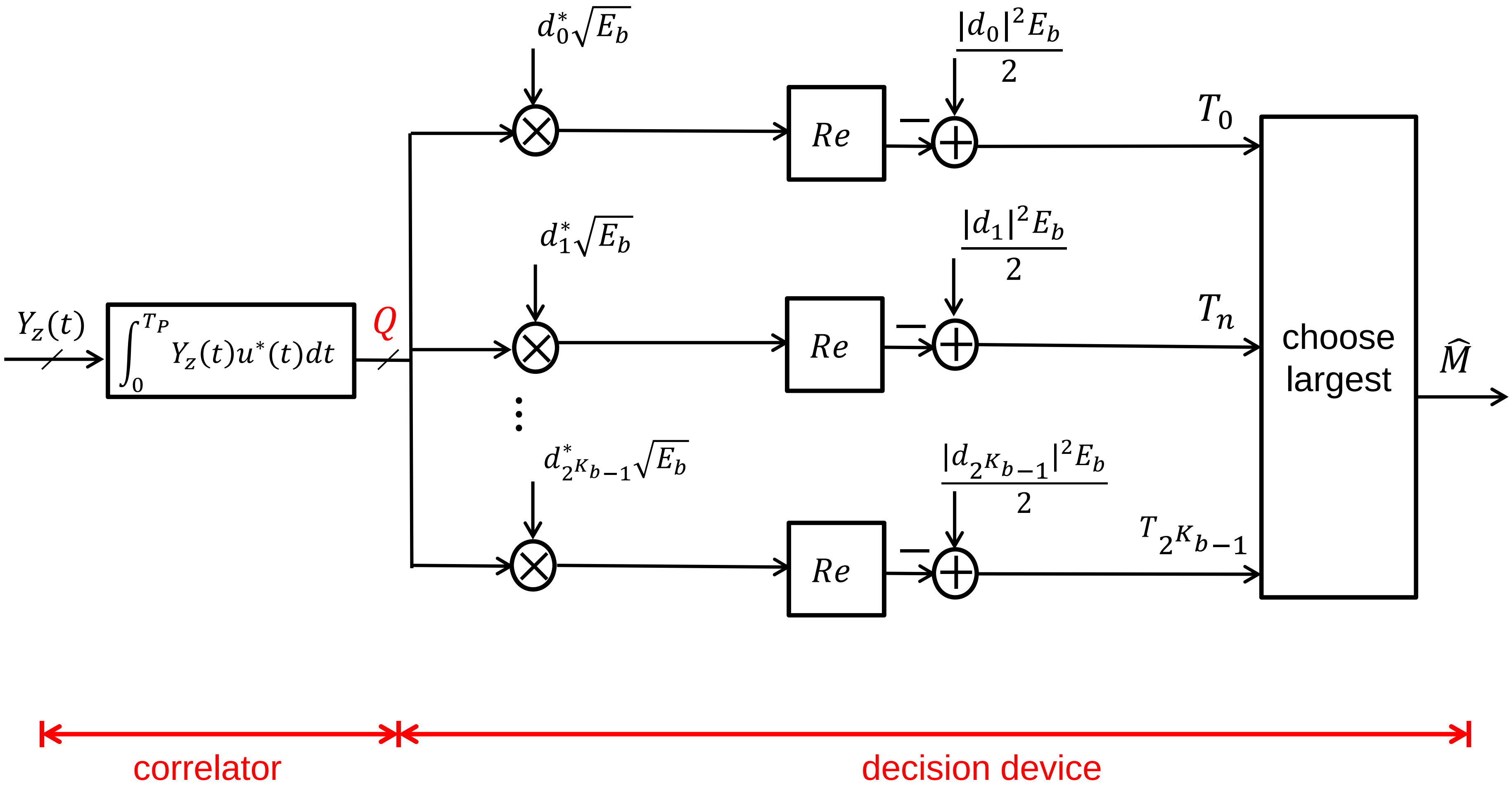
e) 16-QAM ($K_b = 4$)



with A $= \sqrt{\frac{2}{5}}$ $\left(\text{so that } \frac{1}{16}\sum_{i=0}^{15}|d_i|^2 = 4\right)$

Remark: BFSK is not a linear modulation.

- MLWD for linear modulation:



- The decision device can be further simplified.
- The general decision rule is

$$\max_{i \in \{0,\dots,2^{K_b}-1\}} T_i$$

$$= \max_{i \in \{0,\dots,2^{K_b}-1\}} Re\{Qd_i^*\}\sqrt{E_b} - \frac{|d_i|^2 E_b}{2}$$

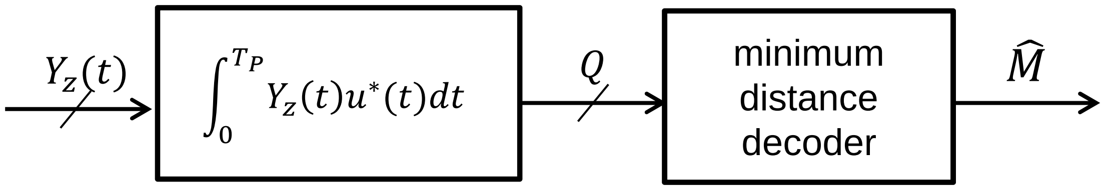
$$= \max_{i \in \{0,\dots,2^{K_b}-1\}} \underbrace{Re\{Qd_i^*\}\sqrt{E_b} - \frac{|d_i|^2 E_b}{2} - \frac{|Q|^2}{2}}_{-\frac{1}{2}\left|Q - d_i\sqrt{E_b}\right|^2}$$

adding a constant that does not depend on i

$$= \min_{i \in \{0,\dots,2^{K_b}-1\}} \left|Q - d_i\sqrt{E_b}\right|^2$$

minimum distance decoder

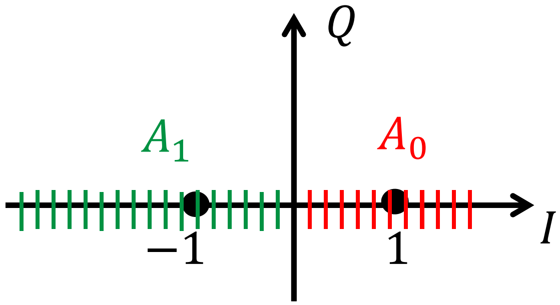
- Therefore, for linear modulations, the MLWD is



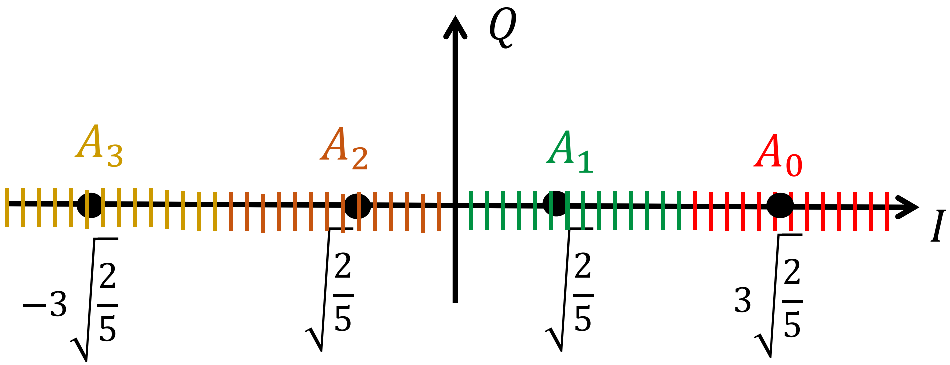
- The minimum distance decoder partitions the complex plane into decision regions A_i, $i \in \{0, \ldots, 2^{K_b} - 1\}$, so that:

$$Q/\sqrt{Eb} \in A_i \Rightarrow \widehat{M} = i$$

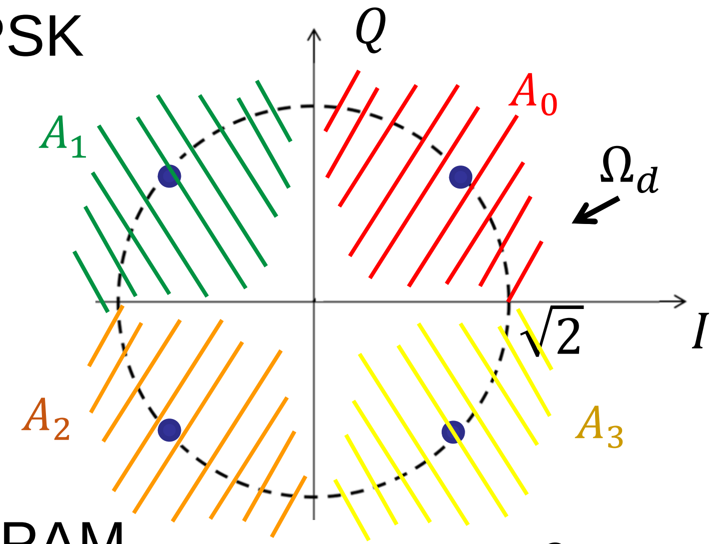
Ex.: a) BPSK



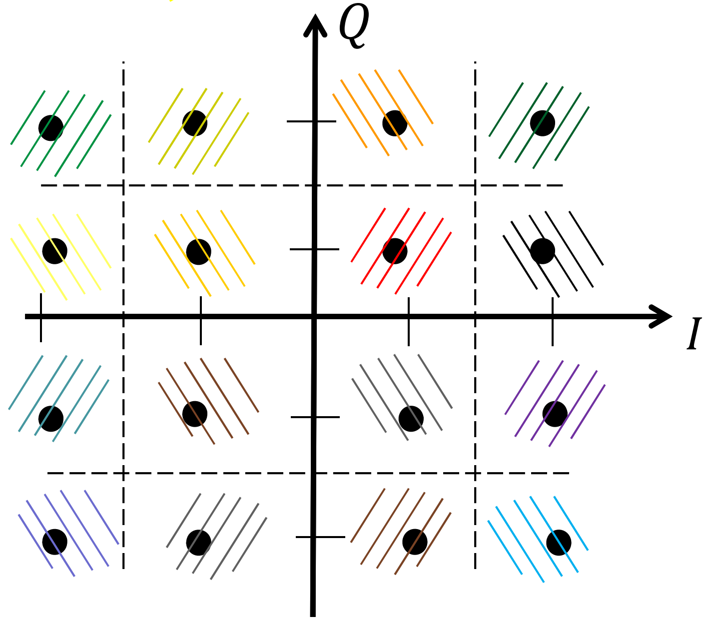
b) 4-PAM



c) 4-PSK



d) 16-PAM



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