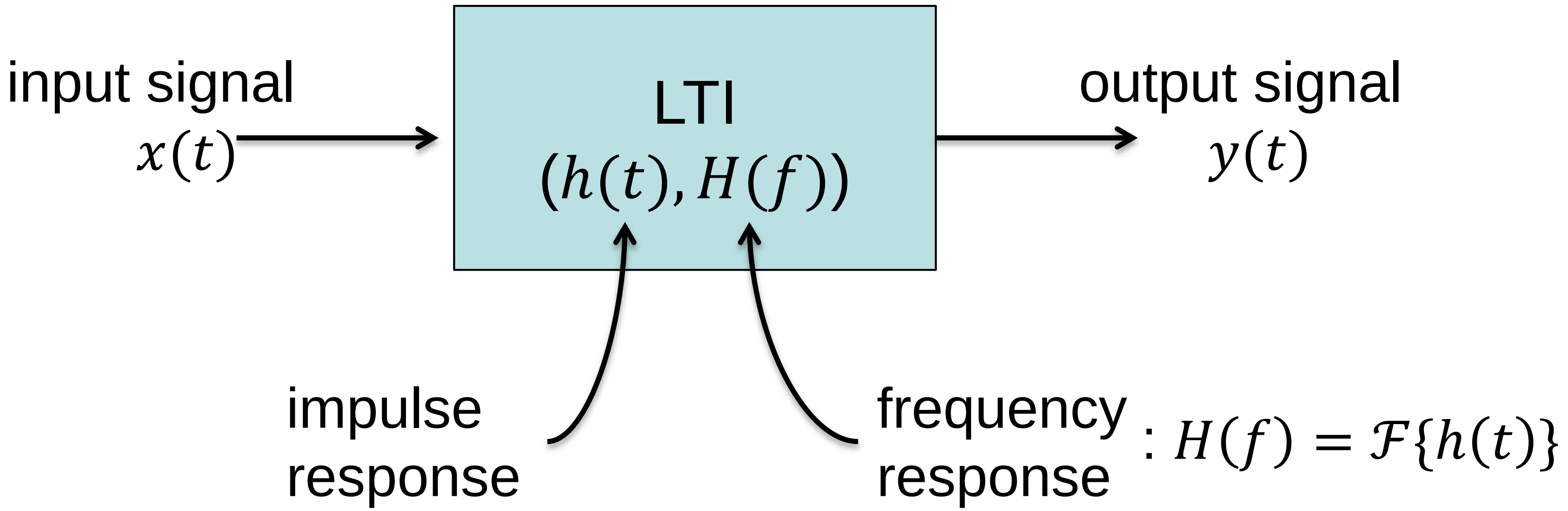
NJIT

New Jersey's Science & Technology University

THE EDGE IN KNOWLEDGE

Linear Time Invariant (LTI) Systems

(Chapter 2: 2.3)



Linear Time Invariant (LTI) Systems

- Time domain:

$$y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(\lambda)h(t-\lambda)d\lambda$$

convolution

$$= \int_{-\infty}^{+\infty} h(\lambda)x(t-\lambda)d\lambda$$

$$= h(t) * x(t)$$

Linear Time Invariant (LTI) Systems

- Frequency domain:

$$Y(f) = H(f)\,X(f)$$

Remark: Energy spectral densities:

$$G_y(f) = |H(f)|^2 G_x\,(f)$$

output energy spectral density

input energy spectral density

Examples

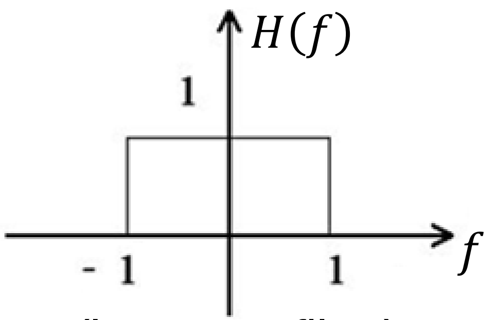
a) $h(t) = \delta(t) \longrightarrow H(f) = 1 \longrightarrow Y(f) = X(f)$

$y(t) = x(t)$

(LTI does not change the input)

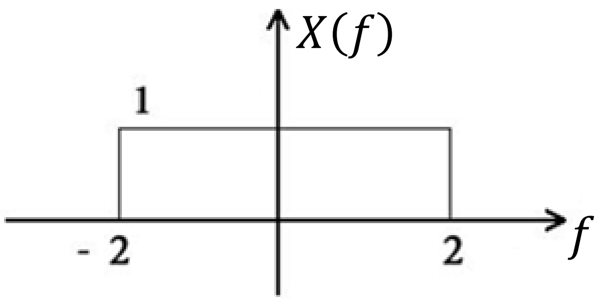
Examples

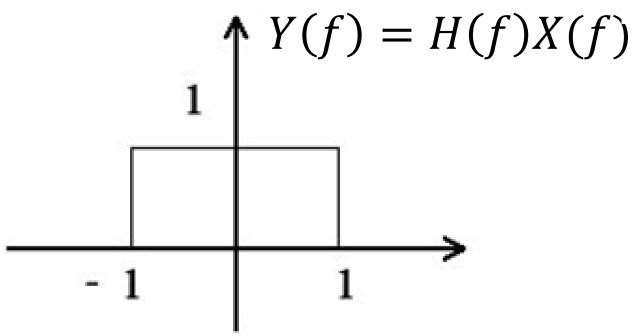
b)



(low-pass filter)

and $x(t) = 4\,\mathrm{sinc}(4t)$

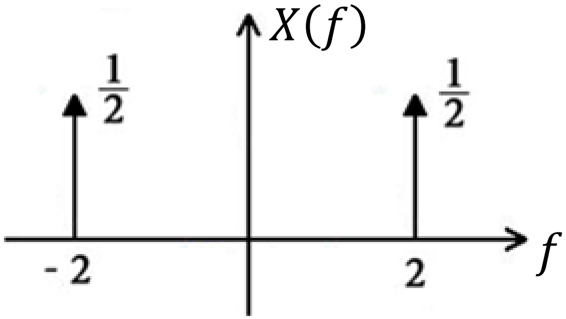




and $y(t) = 2\,\mathrm{sinc}(2t)$

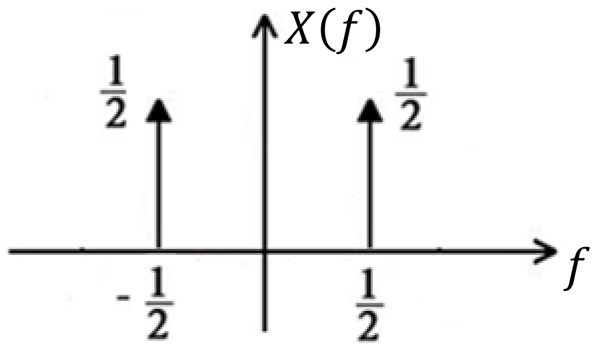
Examples

c) Same low-pass filter as above, but with input $x(t) = \cos(4\pi t)$



$$\rightarrow \begin{cases} Y(f) = H(f)\,X(f) = 0 \\ y(t) = 0 \end{cases}$$

d) Repeat for $x(t) = \cos(\pi t)$



$$\rightarrow \begin{cases} Y(f) = X(f) \\ y(t) = x(t) \end{cases}$$

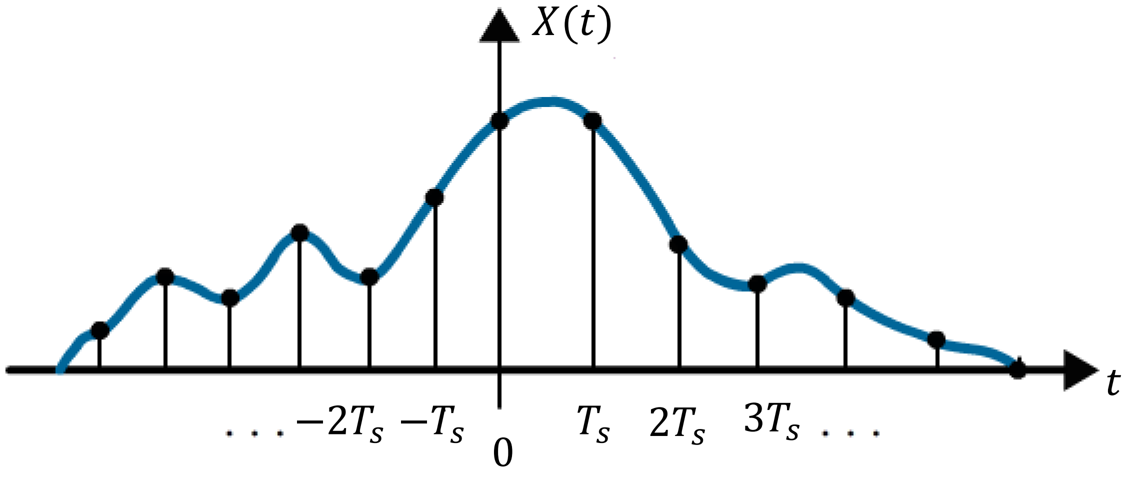
Linear Time Invariant (LTI) Systems

Remark: LTI can be used to implement filters that select only some frequencies of the input signal (e.g., low-pass filter of the previous example).

Using MATLAB (continued)

(Chapter 2: 2.4)

- A signal $x(t)$ is represented as the vector $x = [\dots x(-2T_s), x(-T_s), x(0), x(T_s), x(2T_s), \dots]$ by sampling



Using MATLAB

- How to choose T_s?

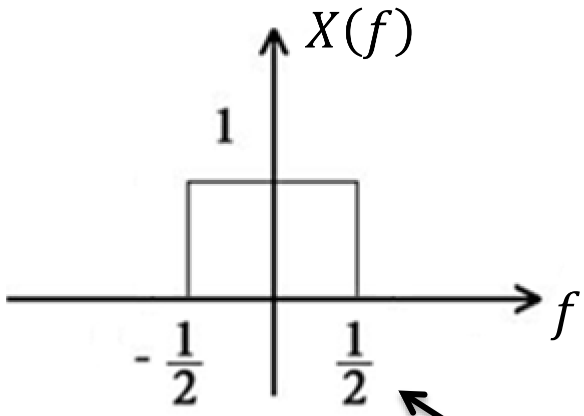
- Nyquist-Shannon theorem:

$$\boxed{\frac{1}{T_s} \geq 2 \times \text{ highest frequency of } X(f)}$$

- In practice, in order to plot in MATLAB, it is suggested to use $\frac{1}{T_s} \geq 10 \times \ (2 \times \text{ highest frequency of } X(f))$

Example

- $x(t) = \mathrm{sinc}(t)$



highest frequency

$$\longrightarrow \frac{1}{T_s} \geq 2 \times \frac{1}{2} = 1$$

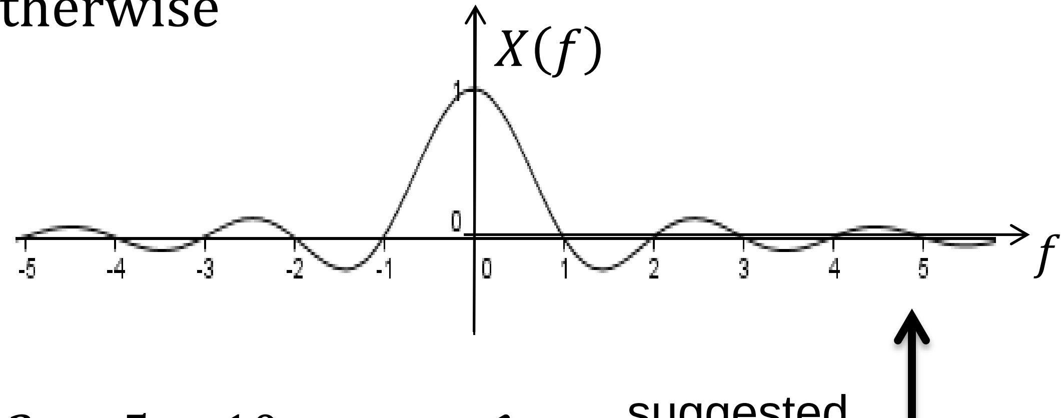
$$\longrightarrow T_s \leq 1$$

Example

- If the highest frequency is infinite, approximate it so that the residual error is small.

Ex.: $x(t) = \begin{cases} 1 & -\frac{1}{2} \le t \le \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$

$$X(f) = \mathrm{sinc}(f)$$



$$\frac{1}{T_s} \ge 2 \times 5 = 10$$ ← suggested approximate highest frequency

$$\rightarrow T_s \le 0.1$$

Example

- To check the effect of T_s, use MATLAB to plot $x(t)$.

  $T_s = 0.1;$ % not a good choice!

  $t = [-1\!:\!T_s\!:\!1];$ % time axis between -1 and 1

  $x = \big((t \geq -0.5)\ \&\ (t \leq 0.5)\big);$ % rectangular signal

  $\text{plot}\ (t, x);$

  $\text{xlabel}\ ('t\ [s]');$

  $\text{ylabel}\ ('x(t)');$

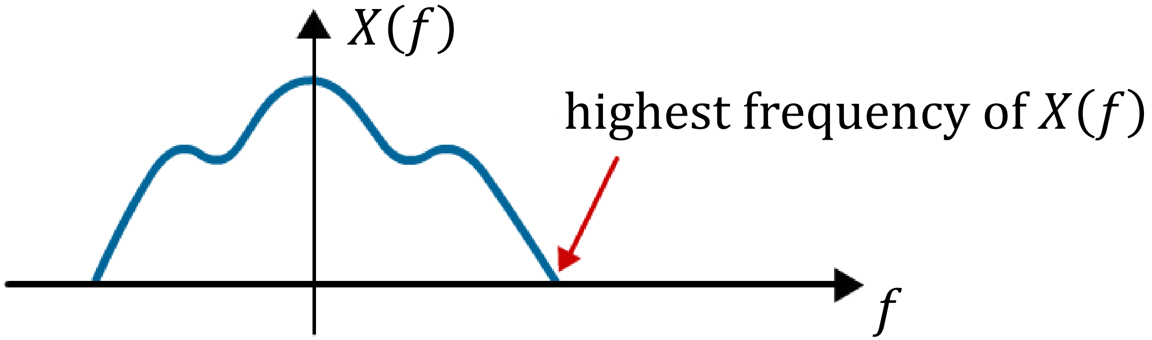
  Repeat with the good choice $T_s = 0.01$.

Using MATLAB

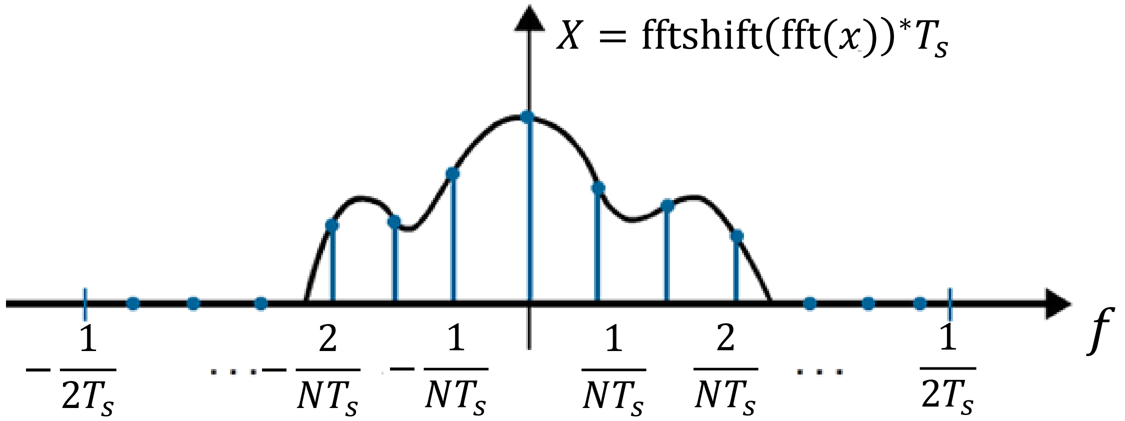
- We can use MATLAB to calculate and plot the Fourier transform.
- The Fourier transform $X(f)$ is represented in MATLAB by a vector X of dimension N equal to the dimension of the signal x in time domain

Using MATLAB

- A Fourier transform



is represented as the *N* dimensional vector

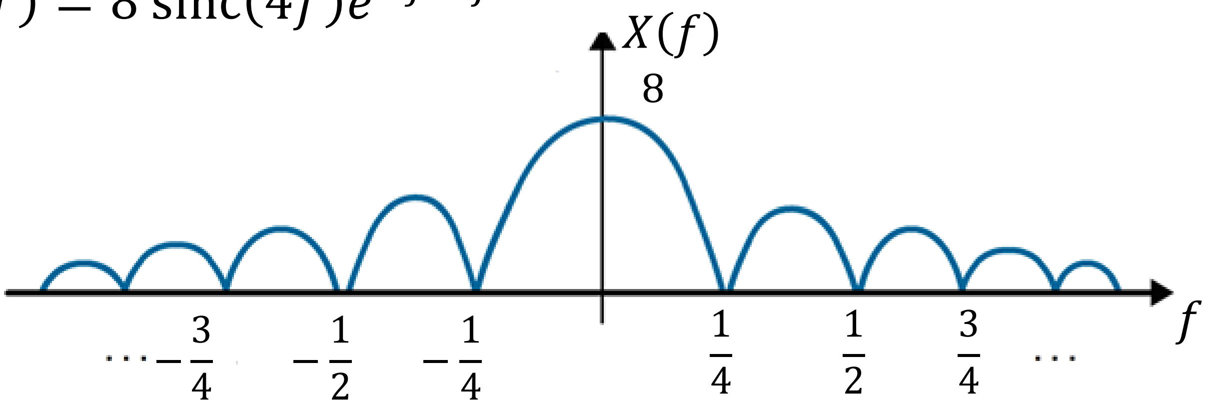


Example

Remark: The picture on the previous slide illustrates that the condition $\frac{1}{T_s} \geq 2 \times \text{highest frequency of } X(f)$ is very important!

Ex.: $x(t) = \begin{cases} 2 & 0 \leq t \leq 4 \\ 0 & \text{elsewhere} \end{cases}$

$X(f) = 8\,\mathrm{sinc}(4f)e^{-j4\pi f}$



We wish to plot $|X(f)|$ using MATLAB

Example

- Choice of T_s

$$\frac{1}{T_s} = 10 \times 2 \times (\text{approximate) highest frequency of } X(f)$$

$$= 10 \times 2 \times \frac{5}{4} = \frac{100}{4} = 25$$

$$\longrightarrow T_s = \frac{1}{25}$$

Example

$T_s = 1/25$; % sampling period

$t = [0{:}T_s{:}100]$; % time axis

$x = 2 * (\, t \leq 4)$;

$\mathrm{N} = \mathrm{length}\ (x)$;

% calculate and plot Fourier transform

$X = \mathrm{fftshift}\ (\mathrm{fft}(x))^* T_s$; % calculate the Fourier transform

Example

$$f = [-1/(2^*T_s) : 1/(N^*T_s) : 1/(2^*T_s) - 1/(N^*T_s)];$$

% frequency axis

$$\text{plot }(f, \text{abs}(X));$$
$$\text{xlabel }('f[Hz]');$$
$$\text{ylabel }('|X(f)|');$$

- You can zoom in to take a better look

Using MATLAB

- We can also approximate integrals in MATLAB. For instance, we can write

$$E_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt \approx \sum_k |x[k]|^2 \cdot T_s$$

approximation

$$= \int_{-\infty}^{+\infty} |X(f)|^2 df \approx \sum_k |X[k]|^2 \cdot \frac{1}{NT_s}$$

Example

- Calculate E_x for previous example in MATLAB

  % estimate energy in time domain

  sum (abs(x).^2) * T_s

  $\longrightarrow$ you will get 16.1600

  % estimate energy in frequency domain

  sum (abs(X).^2)*1/(N* T_s)

  $\longrightarrow$ you will get 16.1600

- The real energy is $E_x = 16$ (The estimate can be improved by decreasing T_s)

Using MATLAB

- From the Fourier transform X we can obtain the signal x in time domain by using

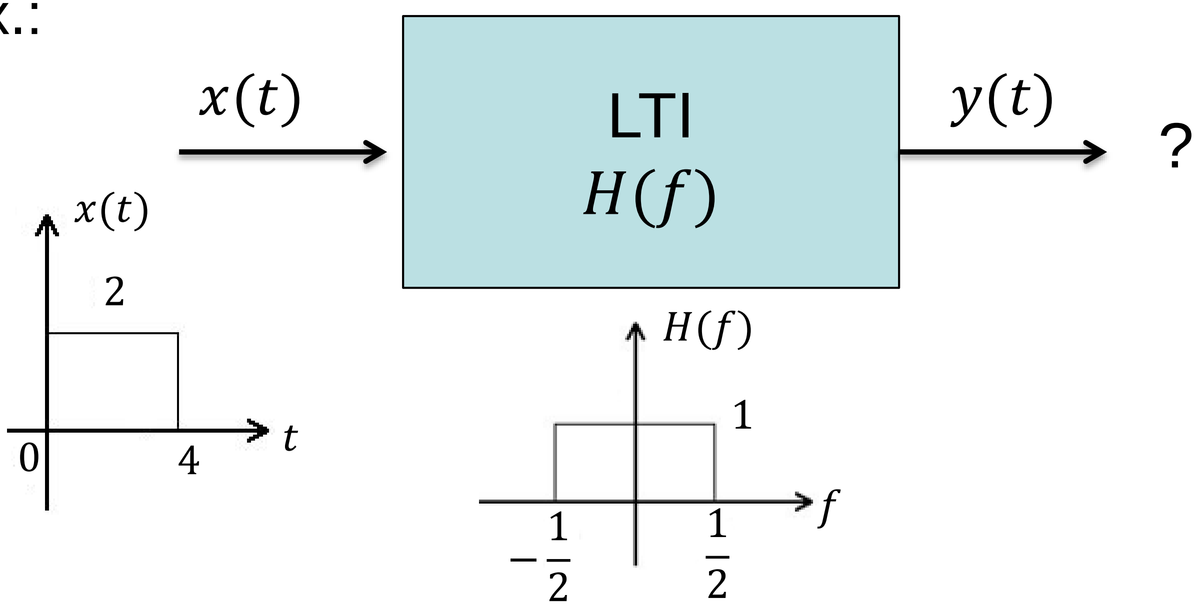
$$x = \mathrm{real}\left(\mathrm{ifft}\left(\mathrm{ifftshift}\left(X\right)\right)\right)*1/T_s \;;$$

(Try in the example)

Using MATLAB

- We can implement an LTI in MATLAB directly in the frequency domain

Ex.:



(low-pass filter)

Example

- Continuing the code of the examples in previous slides

$H = (f \geq -0.5)\ \&\ (f \leq 0.5);$ % frequency response

$Y = H.^*\ X;$ % effect of LTI system (filter) in frequency domain

entry by entry multiplication

$y = \text{real}\ \left(\text{ifft}\ \left(\text{ifftshift}\ (Y)\right)\right)^*1/T_s\ ;$ % output of LTI system in time domain

$\text{plot}\ (t, y);$
$\text{xlabel}\ (t[\text{sec}]');$
$\text{ylabel}\ ('y(t)');$

…What is the effect of the low-pass filter?

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