NJIT

New Jersey's Science & Technology University

THE EDGE IN KNOWLEDGE

Complex Baseband Representation of Bandpass Signals

Part II (Chapter 4: 4.4, 4.5, 4.6)

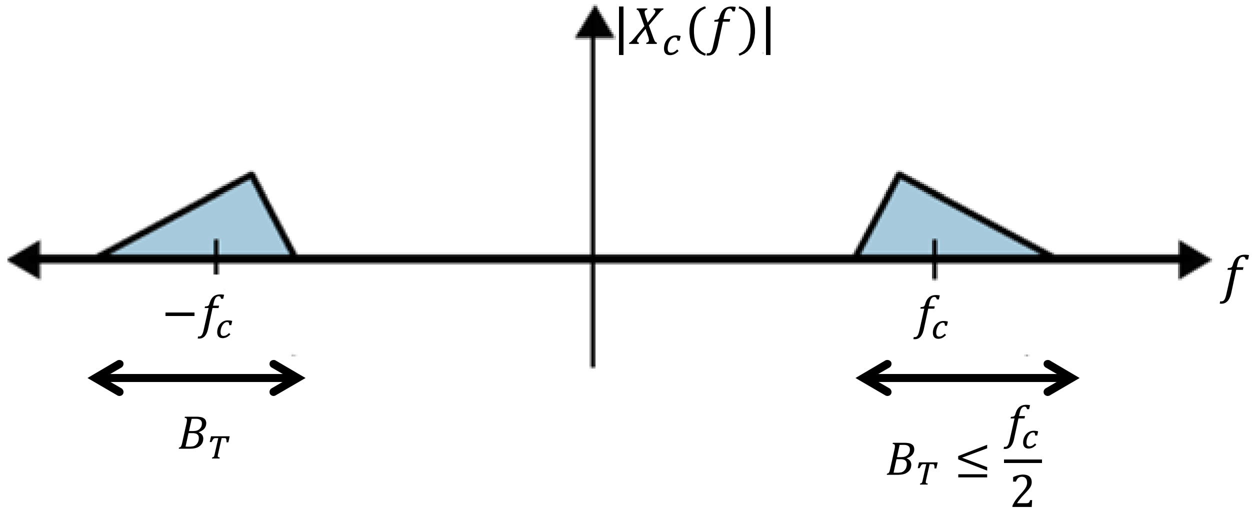
Spectral characteristics of the complex envelope

- As seen, a bandpass signal

$$x_c(t) = \sqrt{2}\, x_I(t)\cos(2\pi f_c t) - \sqrt{2}\, x_Q(t)\sin(2\pi f_c t)$$

is real and hence has a Fourier transform with Hermitian symmetry

Spectral Characteristics of the Complex Envelope



$$\begin{cases} |X_c(f)| = |X_c(-f)| \\ \arg(X_c(f)) = -\arg(X_c(-f)) \end{cases} \Longleftrightarrow \quad X_c(-f) = X_c^*(f)$$

Spectral Characteristics of the Complex Envelope

- We have also seen that it is convenient to represent a bandpass signal $x_c(t)$ using the complex envelope

  $$x_z(t) = x_I(t) + jx_Q(t),$$

  which is complex and baseband

- How to calculate the Fourier transform $X_z(f) = X_I(f) + j\,X_Q(f)$ from $X_c(f)$?

- Remark: $X_z(f)$ needs not satisfy Hermitian symmetry, while $X_I(f)$ and $X_Q(f)$ do (why?)

Spectral Characteristics of the Complex Envelope

- Using the frequency translation property of the Fourier transform, we can calculate

$X_c(f) = \mathcal{F}\{x_c(t)\}$

$$= \sqrt{2}\,\mathcal{F}\{x_I(t)\cos(2\pi f_c t)\} - \sqrt{2}\,\mathcal{F}\{x_Q(t)\sin(2\pi f_c t)\}$$

$$= \frac{1}{\sqrt{2}}(X_I(f-f_c) + X_I(f+f_c))$$

$$- \frac{1}{\sqrt{2}\,j}\left(X_Q(f-f_c) - X_Q(f+f_c)\right)$$

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Spectral Characteristics of the Complex Envelope

$$= \frac{X_I(f - f_c) + jX_Q(f - f_c)}{\sqrt{2}} + \frac{X_I(f + f_c) - jX_Q(f + f_c)}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} X_z(f - f_c) + \frac{1}{\sqrt{2}} X_z^*(-f - f_c)$$

where the last equality follows from the Hermitian symmetry of $X_I(f)$ and $X_Q(f)$, since

$$\begin{aligned} X_c^*(-f - f_c) &= X_I^*(-f - f_c) - j\, X_Q^*(-f - f_c) \\ &= X_I(f + f_c) - j\, X_Q(f + f_c) \end{aligned}$$

Spectral Characteristics of the Complex Envelope

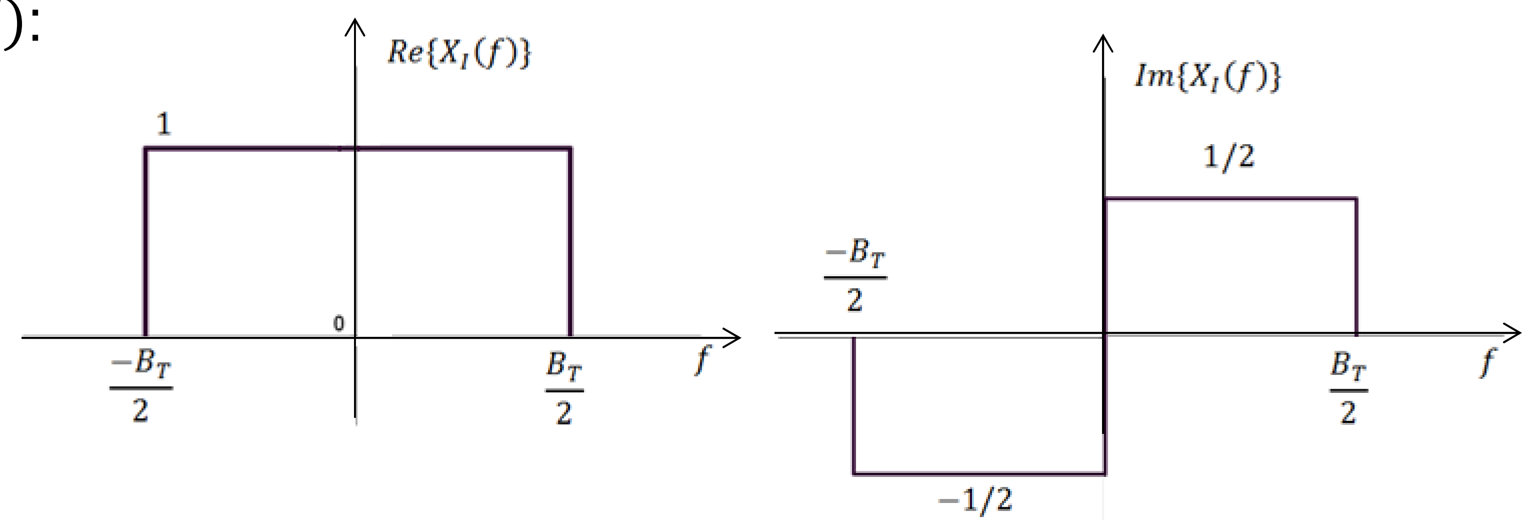
- We have shown that

$$X_c(f) = \frac{1}{\sqrt{2}} X_z(f - f_c) + \frac{1}{\sqrt{2}} X_z^*(-f - f_c)$$

- Remark: $X_c(f)$ satisfies Hermitian symmetry

Example

$X_I(f)$:



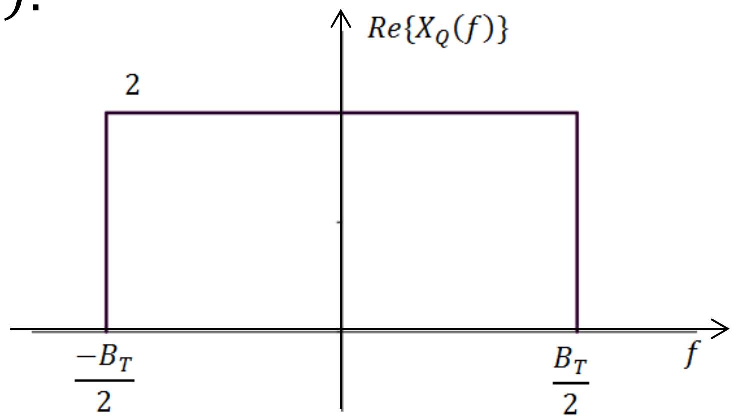
(satisfies Hermitian symmetry:

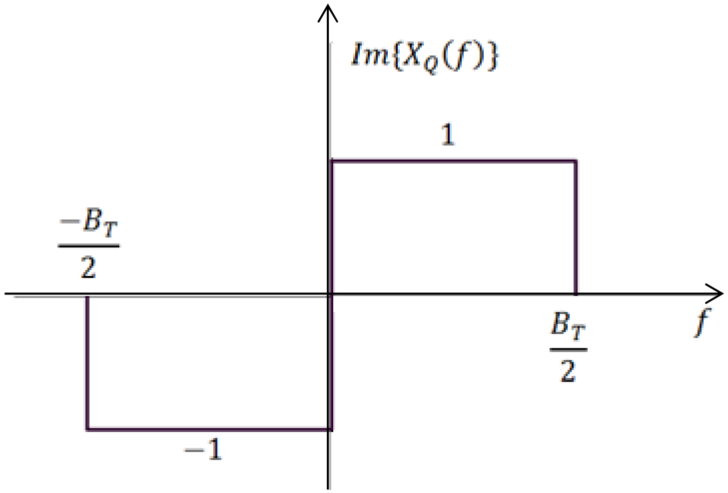
$$Re\{X_I(f)\} = Re\{X_I(-f)\}$$

and $Im\{X_I(f)\} = -Im\{X_I(-f)\}$)

Example

$X_Q(f)$:





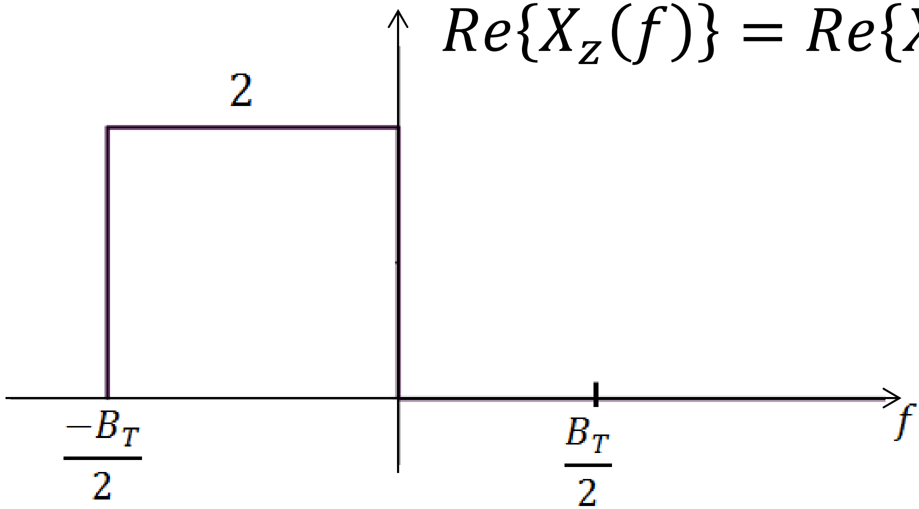
(satisfies Hermitian symmetry:

$$Re\{X_Q(f)\} = Re\{X_Q(-f)\}$$

and $Im\{X_Q(f)\} = -Im\{X_Q(-f)\}$)

Example

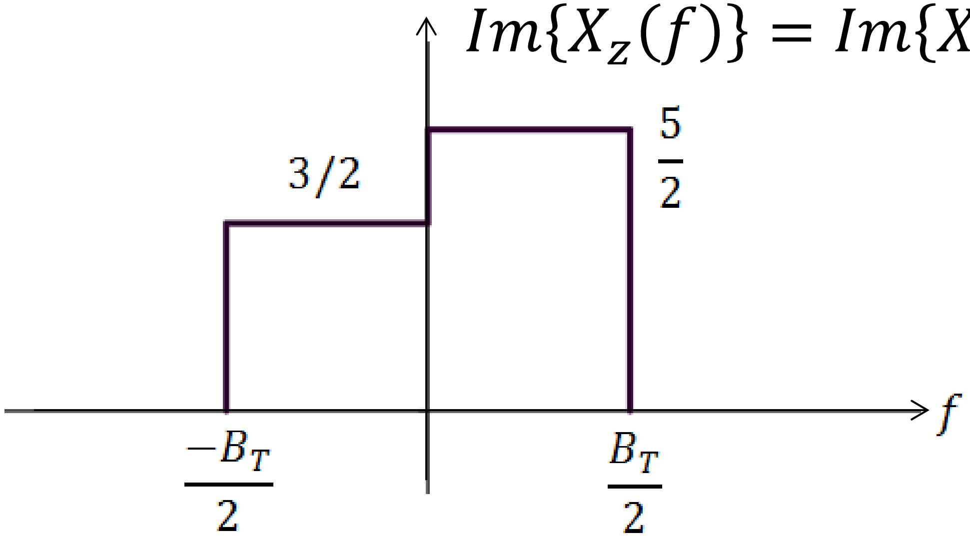
$$Re\{X_z(f)\} = Re\{X_I(f)\} - Im\{X_Q(f)\}$$



(Does not satisfy Hermitian symmetry)

Example

$$Im\{X_z(f)\} = Im\{X_I(f)\} + Re\{X_Q(f)\}$$

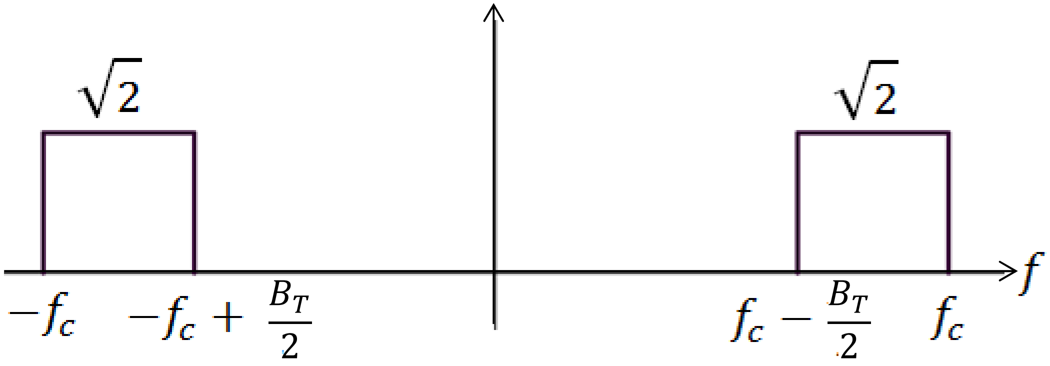


(Does not satisfy Hermitian symmetry)

Example

$$X_c(f) = \frac{X_z(f - f_c)}{\sqrt{2}} + \frac{X_z^*(-f - f_c)}{\sqrt{2}}:$$

$$Re\{X_c(f)\} = \frac{Re\{X_z(f - f_c)\}}{\sqrt{2}} + \frac{Re\{X_z(-f - f_c)\}}{\sqrt{2}}$$

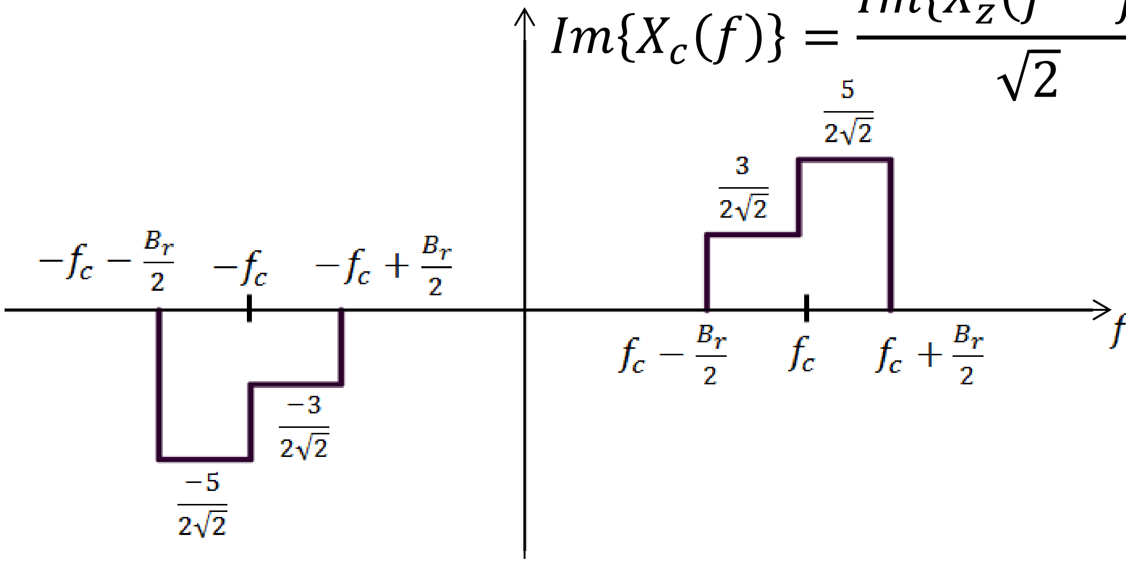


(Satisfies Hermitian symmetry)

Example

$$X_c(f) = \frac{X_z(f - f_c)}{\sqrt{2}} + \frac{X_z^*(-f - f_c)}{\sqrt{2}}:$$

$$Im\{X_c(f)\} = \frac{Im\{X_z(f - f_c)\}}{\sqrt{2}} - \frac{Im\{X_z(-f - f_c)\}}{\sqrt{2}}$$



(Satisfies Hermitian symmetry)

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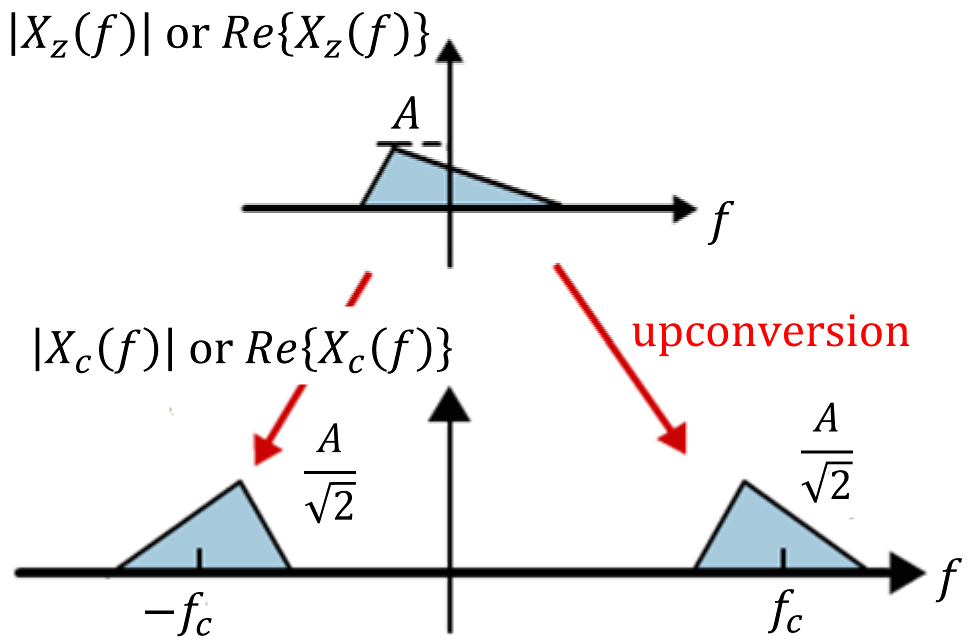
Spectral Characteristics of the Complex Envelope

- Essentially, the relationship between $X_z(f)$ and $X_c(f)$ consists of an upconversion operation that guarantees Hermitian symmetry.



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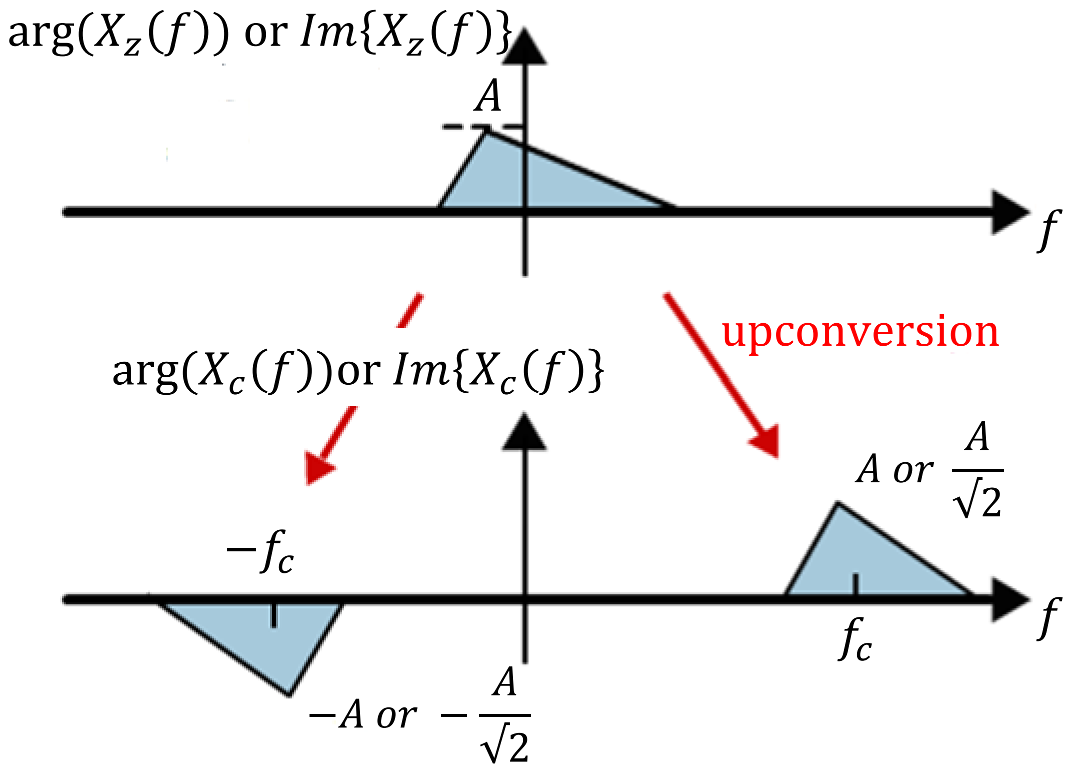
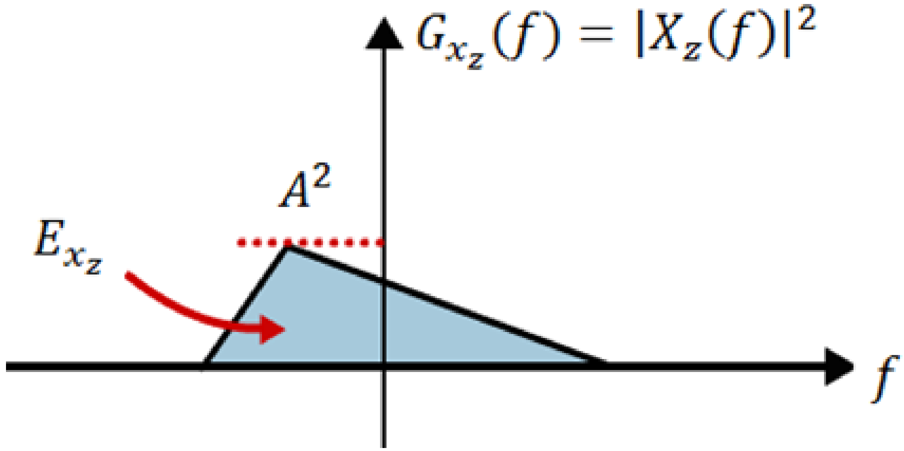
Spectral Characteristics of the Complex Envelope

and



Spectral Characteristics of the Complex Envelope

- Remark: The energy of $x_c(t)$ is the same as the energy of $x_z(t)$. This can be seen using Rayleigh theorem, as illustrated below.



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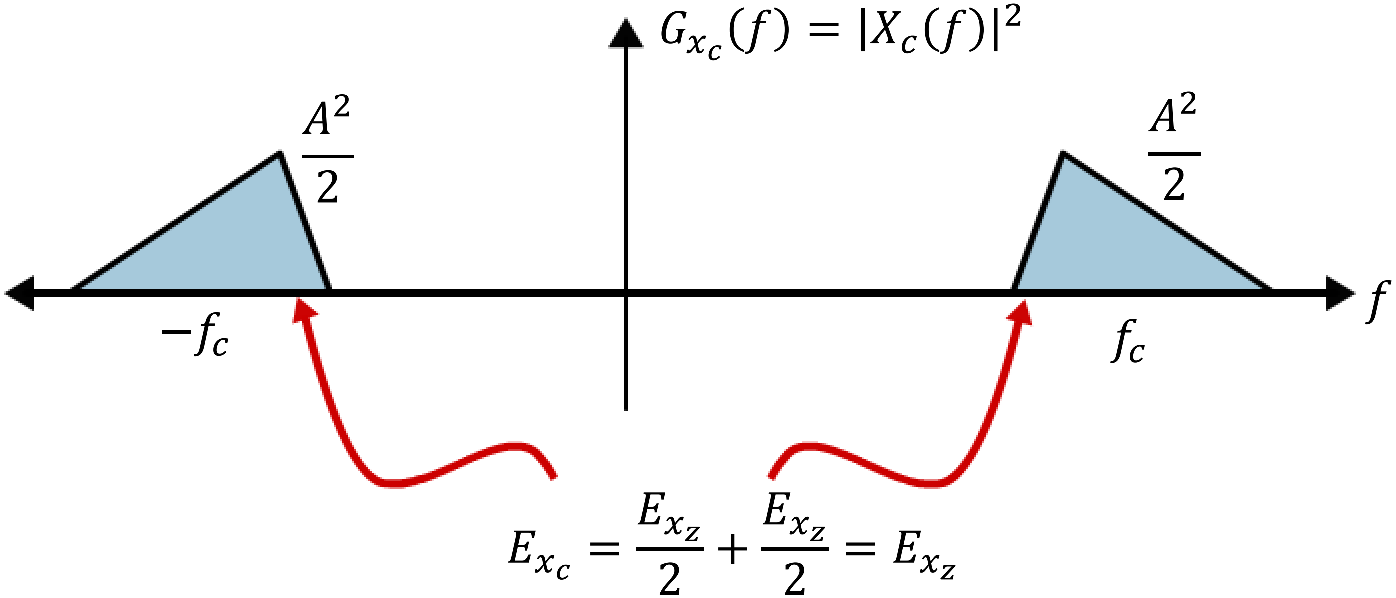
Spectral Characteristics of the Complex Envelope

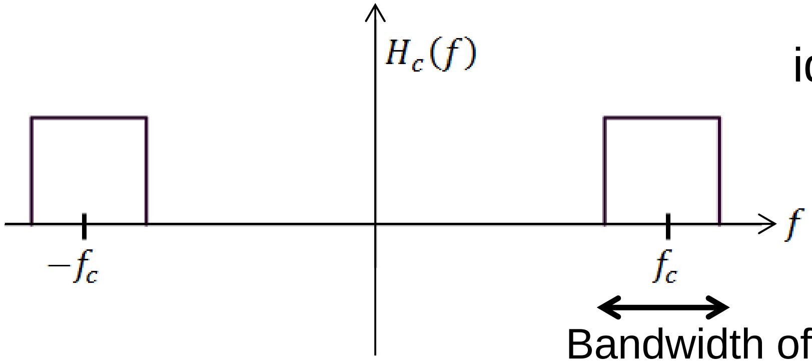


…This explains the term $\sqrt{2}$ in the definition of $x_c(t)$.

LTI Systems and Bandpass Signals

- Filters are extremely important in communication systems

Ex.: Radio transmitters must make sure that the transmitted signals do not interfere with adjacent radio stations.



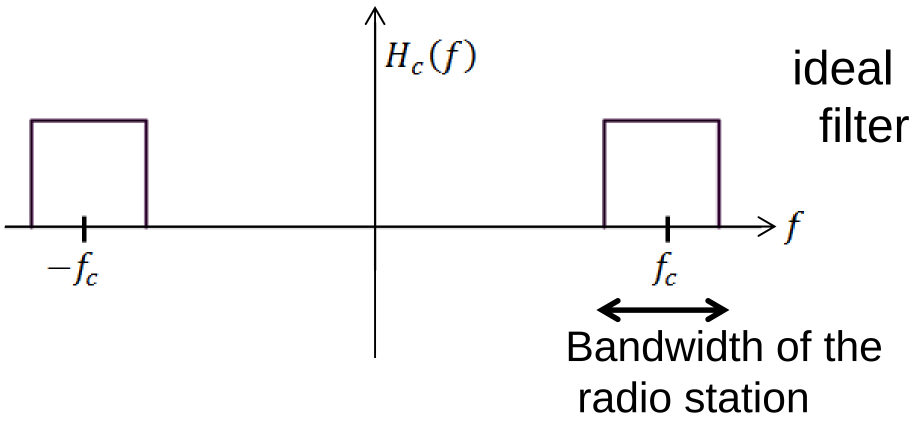
ideal filter

Bandwidth of the radio station

LTI Systems and Bandpass Signals

- Filters are extremely important in communication systems

Ex.: Radio receivers must tune in to the desired radio station while rejecting interference from adjacent radio stations.



LTI Systems and Bandpass Signals

- Bandpass filters:

$$x_c(t) = \sqrt{2}Re\{x_z(t)e^{j2\pi f_c t}\}$$

input bandpass signal with complex envelope $x_z(t)$

Bandpass LTI

$H_c(f)$ or $h_c(t)$

bandpass frequency response

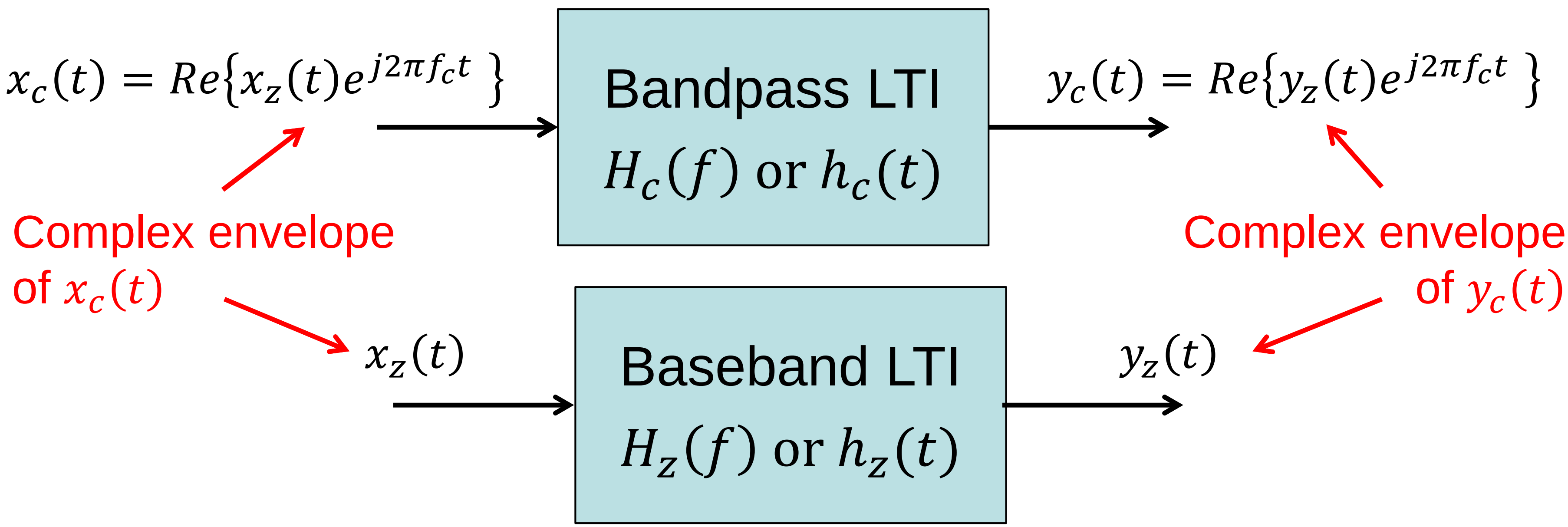
bandpass impulse response

$$y_c(t) = \sqrt{2}Re\{y_z(t)e^{j2\pi f_c t}\}$$

output bandpass signal with complex envelope $y_z(t)$

LTI Systems and Bandpass Signals

- We wish to represent the bandpass filter $H_c(f)$, or $h_c(t)$, with a baseband complex filter as follows:



LTI Systems and Bandpass Signals

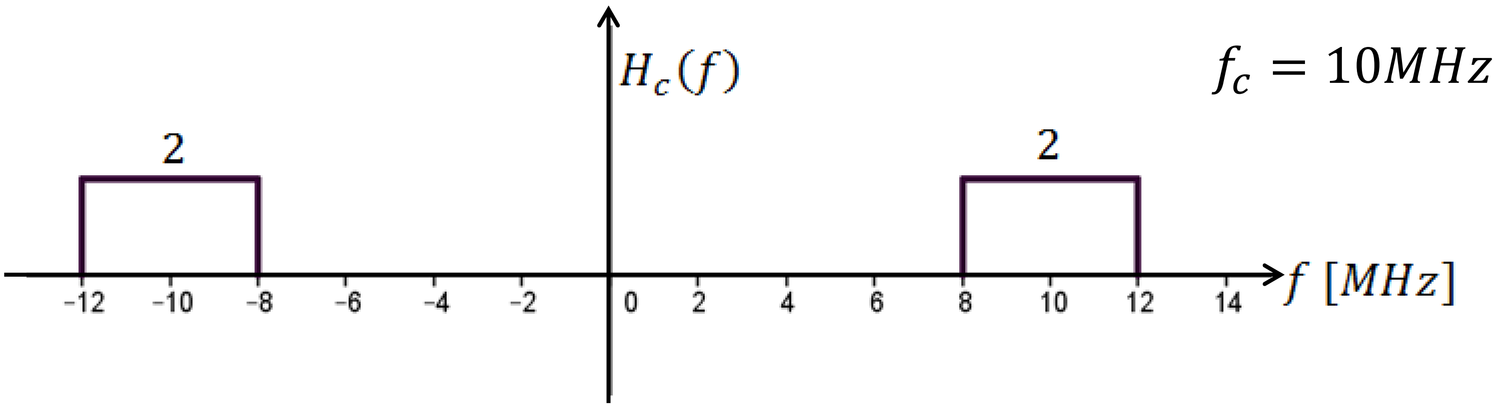
- How to choose the baseband LTI so that we have the equivalence at the previous slide?
- It can be easily seen that we need

$$H_c(f) = H_z(f - f_c) + H_z^*(-f - f_c)$$

- Remark: Unlike for signals, there is no $\frac{1}{\sqrt{2}}$ term.

Example

- Consider the bandpass filter in the figure below.



a) If the input signal is

$$x_c(t) = \sqrt{2}\ x_I(t)\cos(2\pi 10^7 t)$$

with $x_I(t) = \cos(2\pi 10^6 t) + \cos(6\pi 10^6 t)$

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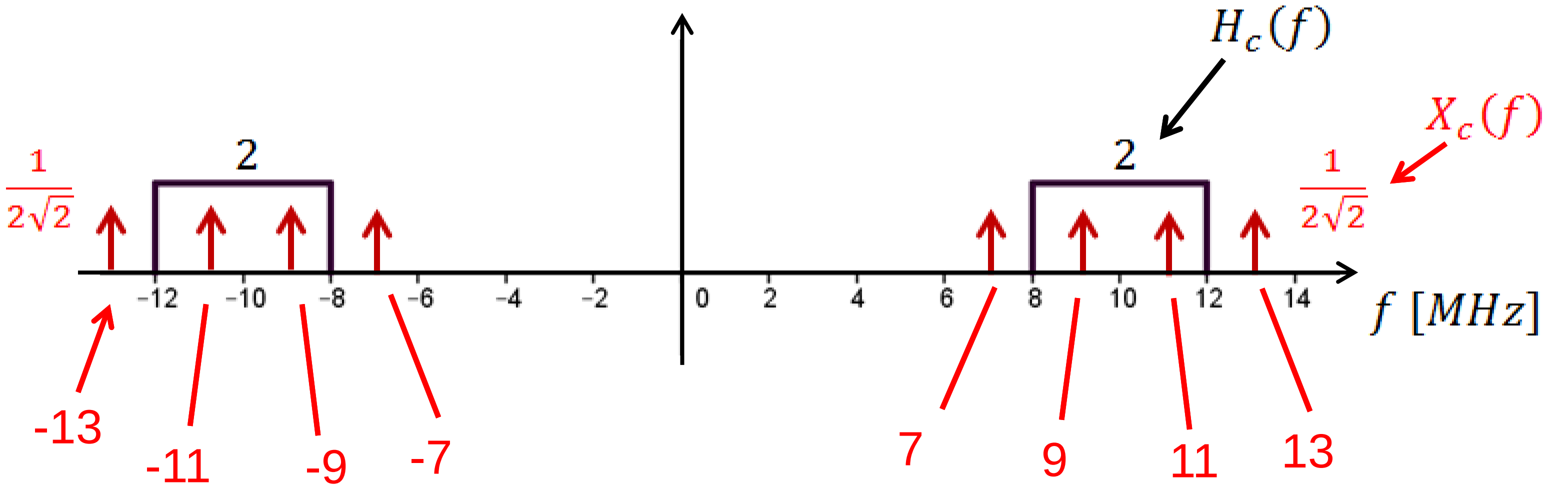
Example

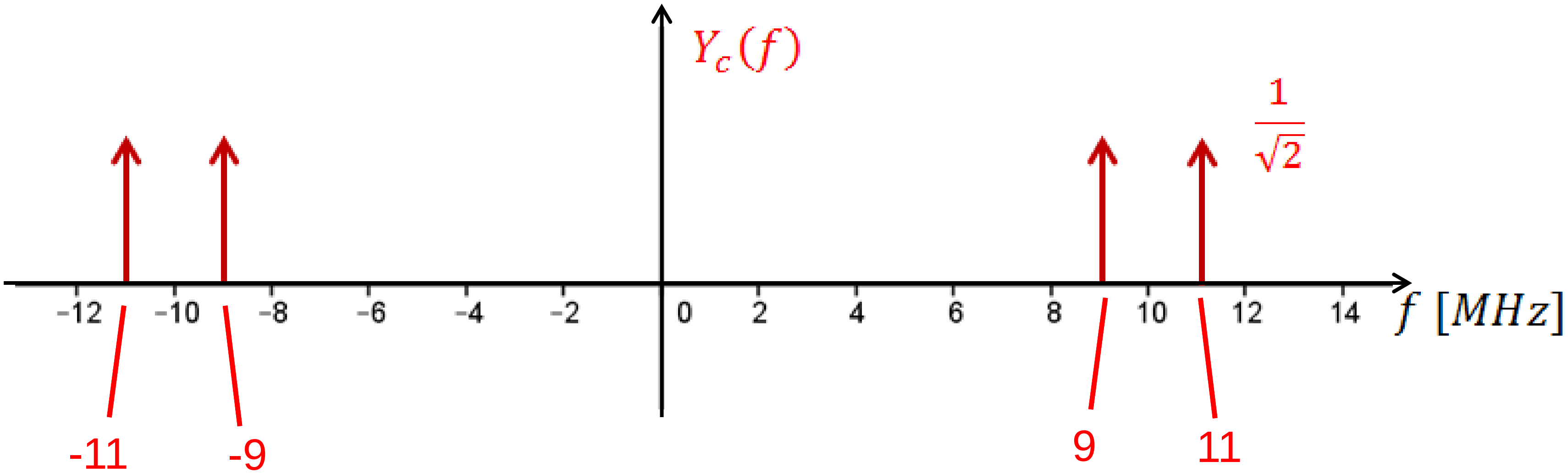
find the output $y_c(t)$:

Input and filter:



Example

Output:



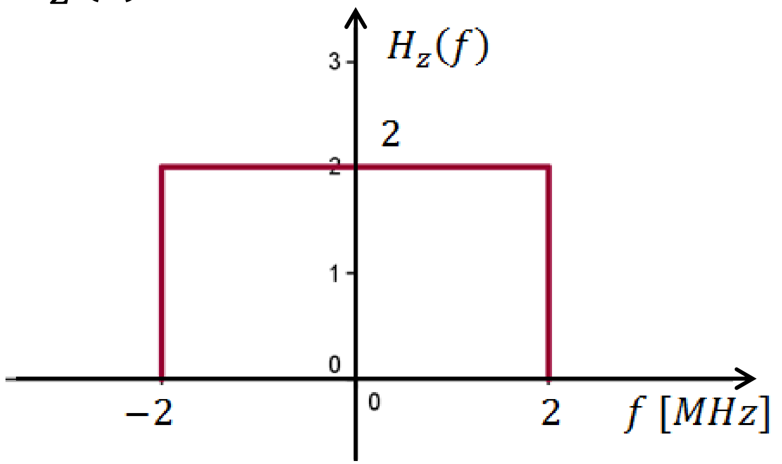
$$\Rightarrow y_c(t) = \sqrt{2}\ \underbrace{(2\cos(2\pi 10^6 t))}_{y_I(t) = y_z(t)} \cos(2\pi 10^7 t)$$

Example

b) Calculate the complex envelope of the output $y_c(t)$:

$$y_z(t) = 2\cos(2\pi 10^6 t)$$

c) Calculate the equivalent baseband filter $H_z(f)$ and $h_z(t)$:

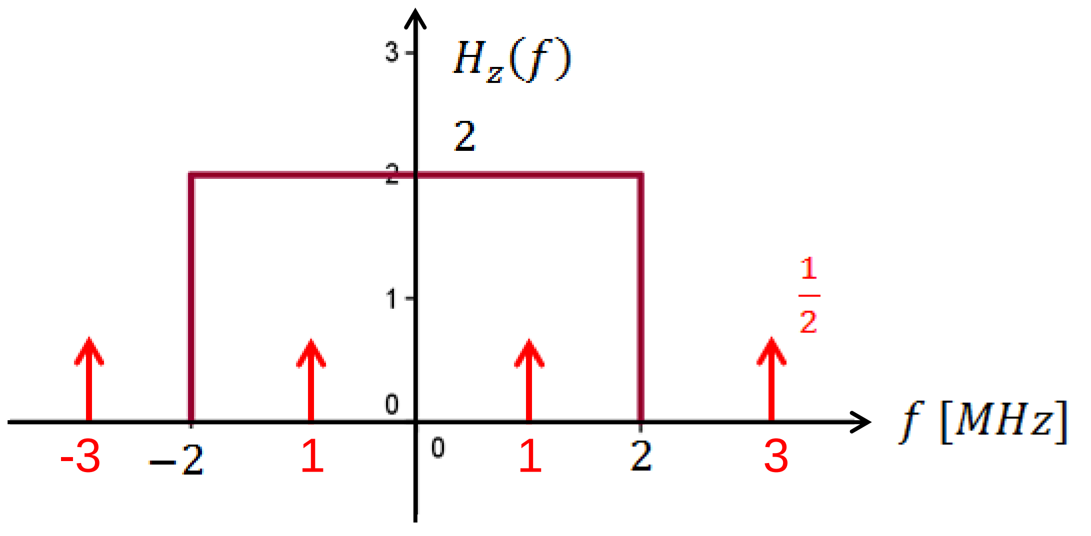


$$\Rightarrow h_z(t) = 8 \times 10^6 \times \mathrm{sinc}(4 \times 10^6 t)$$

Example

d) Using the baseband filter $h_z(t)$, calculate $y_z(t)$. Compare with the results at point b).

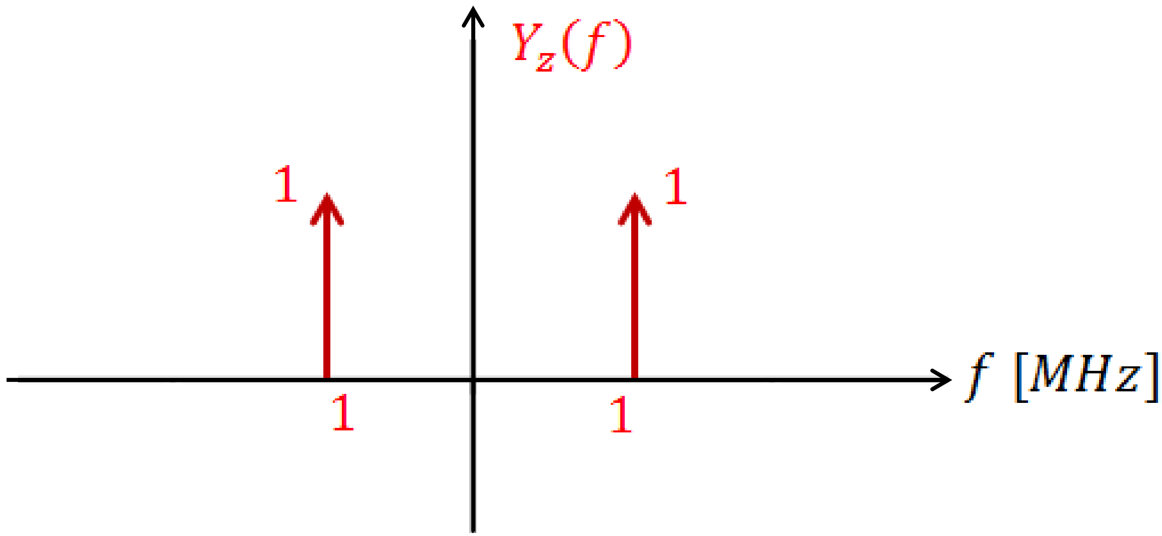
Input and filter:



Example

d) Using the baseband filter $h_z(t)$, calculate $y_z(t)$. Compare with the results as at point b).

Output:



$\Rightarrow y_z(t) = 2\cos\left(2\pi 10^6 t\right)$, as at point b).

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