NJIT

New Jersey's Science & Technology University

THE EDGE IN KNOWLEDGE

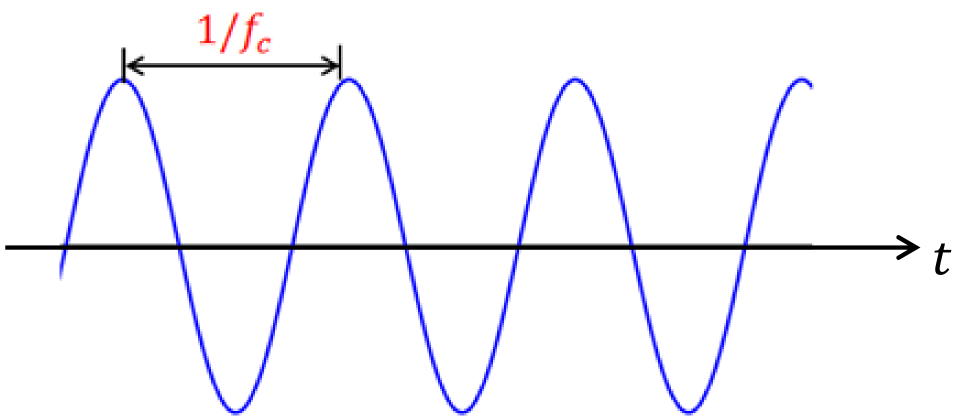
Complex Baseband Representation of Bandpass Signals

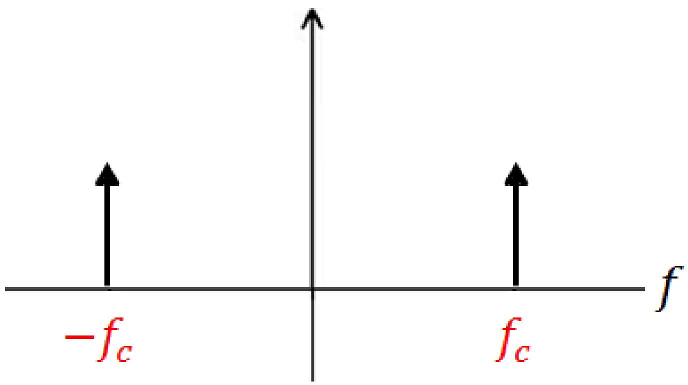
Part 1 (Chapter 4: 4.1, 4.2, 4.3)

- Most communication systems operate by <u>modulating</u> information onto a <u>sinusoidal carrier</u>.

- Sinusoidal carrier: $\cos(2\pi f_c\, t + \vartheta)$

carrier frequency





Example

System	Carrier Frequency f_c
AM radio	530-1600 kHz
FM radio	88-108 MHz
Cellular	~900 MHz, ~1-2 GHz
Wi-Fi	2.4 GHz
Satellite	~3-6 GHz
Fiber optics	200 THz

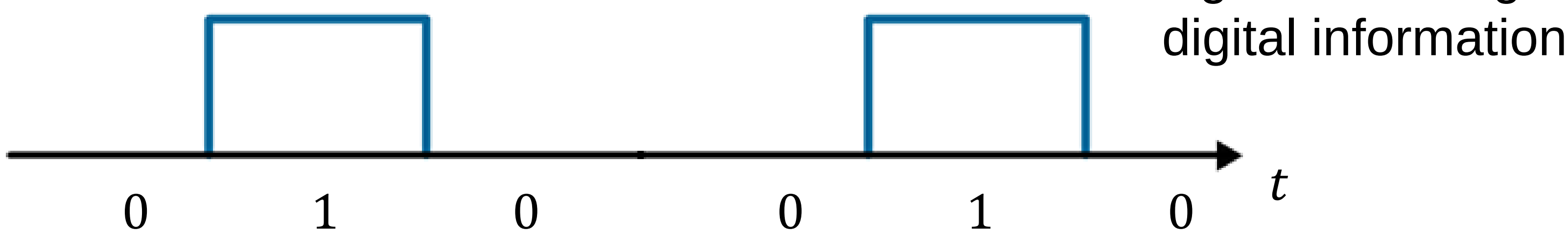
legend: $k \rightarrow 10^3, M \rightarrow 10^6, G \rightarrow 10^9, T \rightarrow 10^{12}$

$Hz = \text{cycles}/s$

Complex Baseband Representation of Bandpass Signals

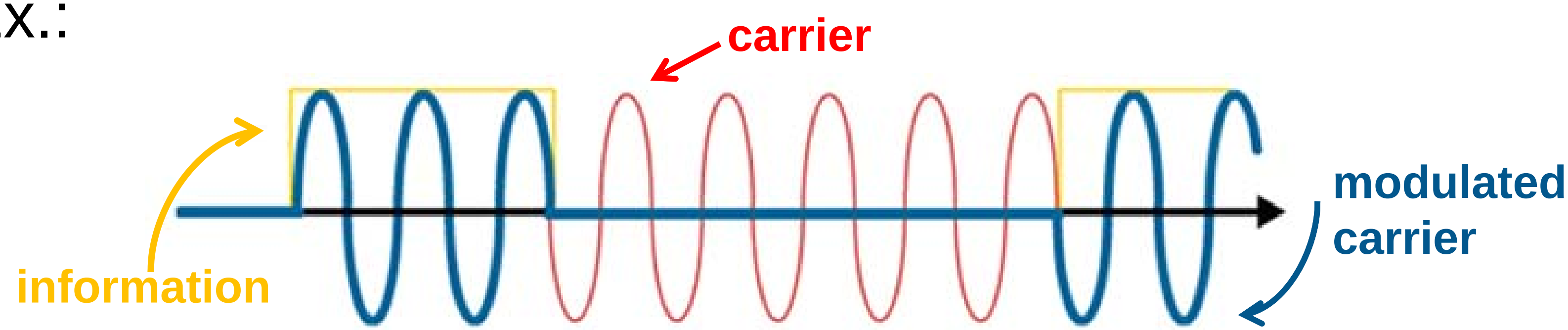
- Information can be analog or digital

Ex.:

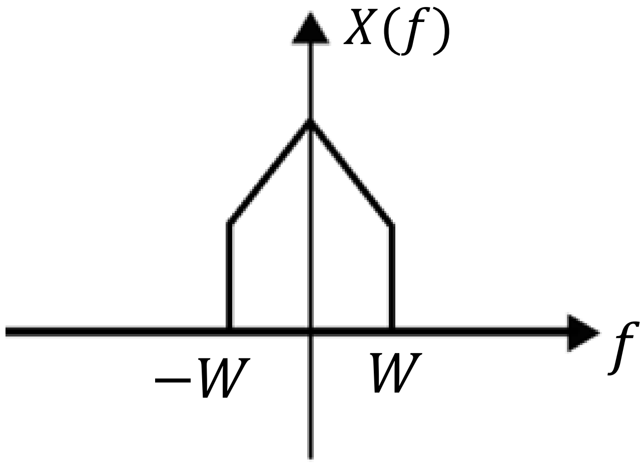


- The information signal modulates a sinusoidal carrier

Ex.:

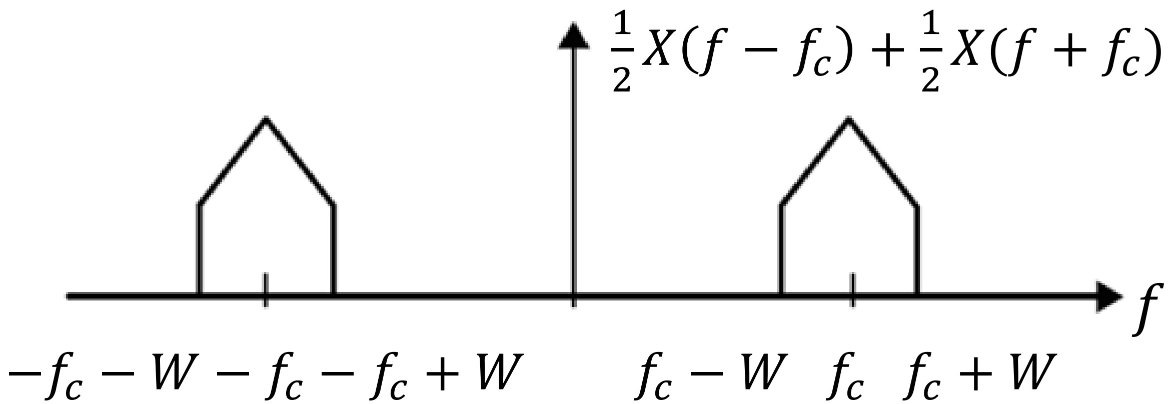


Example: Amplitude Modulation



information signal: $x(t)$

Baseband Signal



amplitude-modulated signal:
$x(t)\cos(2\pi f_c t)$

Bandpass Signal

Why do we need modulation?

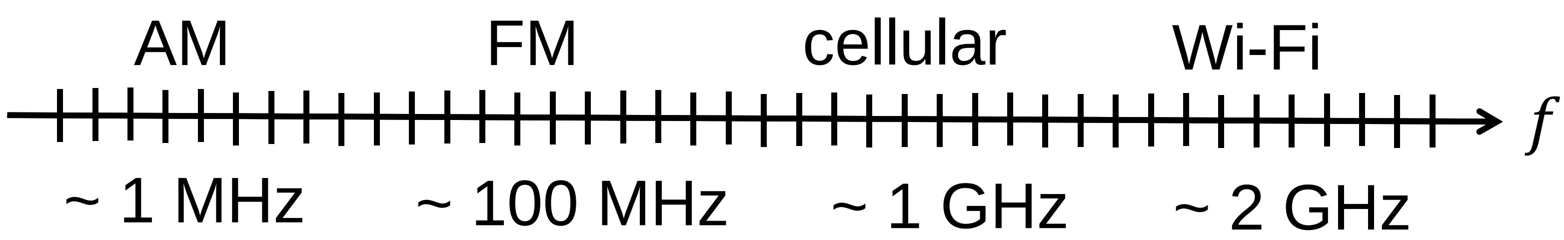
1. The antenna size depends on the wavelength
   $\lambda = \frac{c}{f_c} \quad (c = 3 \times 10^8 \text{ m/s})$

System	Wavelength λ
AM	~ 300 m
FM	~ 3 m
Cellular	~ 0.3 m
Wi-Fi	~ 0.1 m

$\longrightarrow$ need modulation prior to transmission

Why do we need modulation?

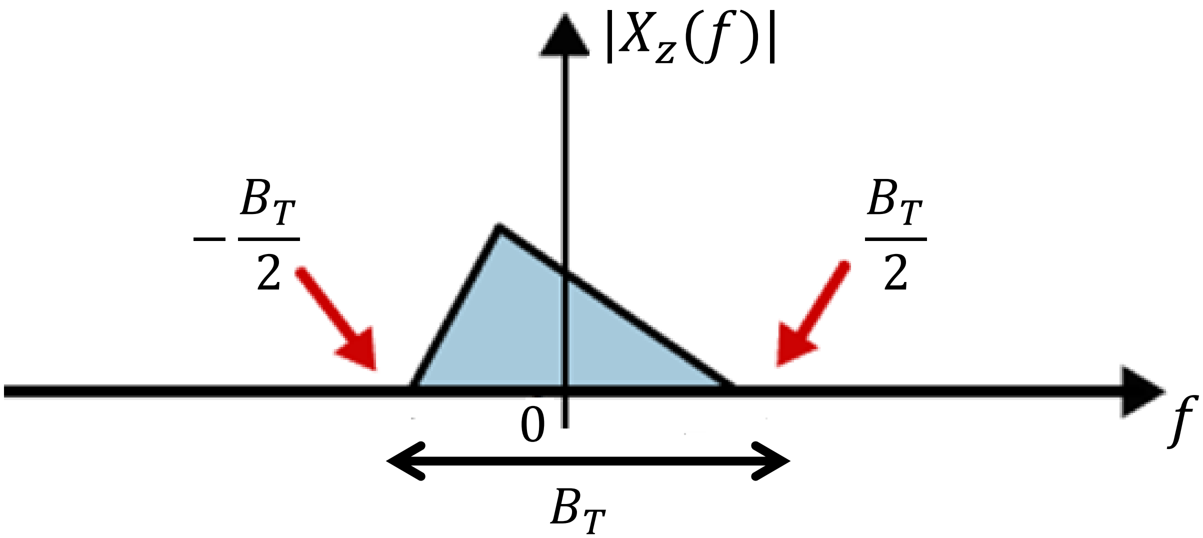
2. Frequency Division Multiplexing (FDM)



…different information streams (e.g., radio stations) modulated on different carriers.

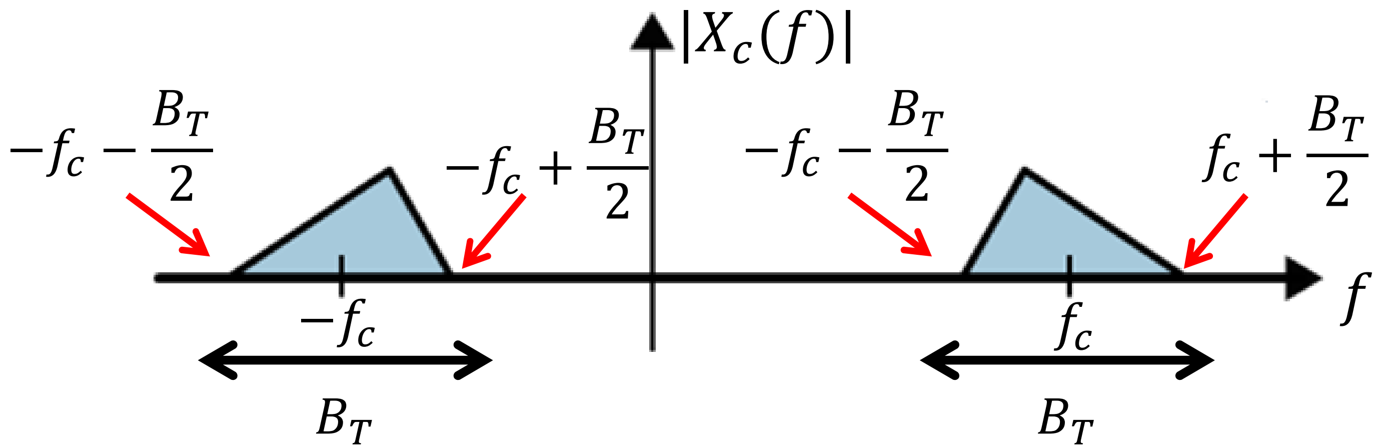
Complex Baseband Representation of Bandpass Signals

- Baseband signal $x_z(t)$: Fourier transform is non-zero only in a bandwidth B_T around the zero frequency



Complex Baseband Representation of Bandpass Signals

- Bandpass signal $x_c(t)$: Fourier transform is non-zero only in a bandwidth B_T around the carrier frequency $\pm f_c$ $\left(\frac{B_T}{2} < f_c\right)$



Complex Baseband Representation of Bandpass Signals

Ex.:

System	Bandwidth B_T
AM	300 kHz
FM	180 kHz
Cellular	1-20 MHz
Wi-Fi	20 MHz

Complex Baseband Representation of Bandpass Signals

- A baseband signal $x_z(t)$ is either real or complex in this course
- A bandpass signal $x_c(t)$ is always real in this course

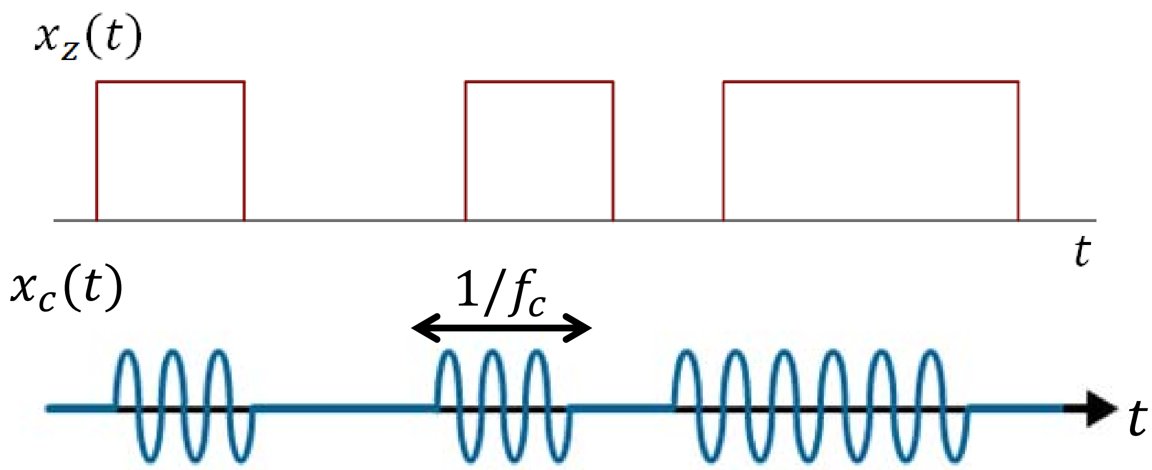
Hermitian symmetry $\rightarrow$

$$\begin{cases} |X_c(f)| = |X_c(-f)| \text{ or } G_x(f) = G_x(-f) \\ \arg(X_c(f)) = -\arg(X_c(-f)) \end{cases}$$

Complex Baseband Representation of Bandpass Signals

- In this course
  - baseband signals are information signals
  - passband signals are modulated signals

Ex.:



baseband information signal

passband modulated signal

Complex Baseband Representation of Bandpass Signals

- Remark: In the example on the previous slides, we have

$$x_c(t) = x_z(t)\cos(2\pi f_c t).$$

We will call this type of modulation Amplitude Modulation, since the baseband information signal modulates the amplitude of the carrier.

As we will see, amplitude modulation is not the only type of modulation.

Complex Baseband Representation of Bandpass Signals

- Mathematically, a bandpass signal is defined as:

$$x_c(t) = \sqrt{2}\, \underbrace{x_A(t)}_{\substack{\text{amplitude} \\ \text{(non-negative)}}} \cos(2\pi f_c t + \underbrace{x_P(t)}_{\text{phase}})$$

→ Information is encoded by the two baseband signals $x_A(t)$ and $x_P(t)$

Complex Baseband Representation of Bandpass Signals

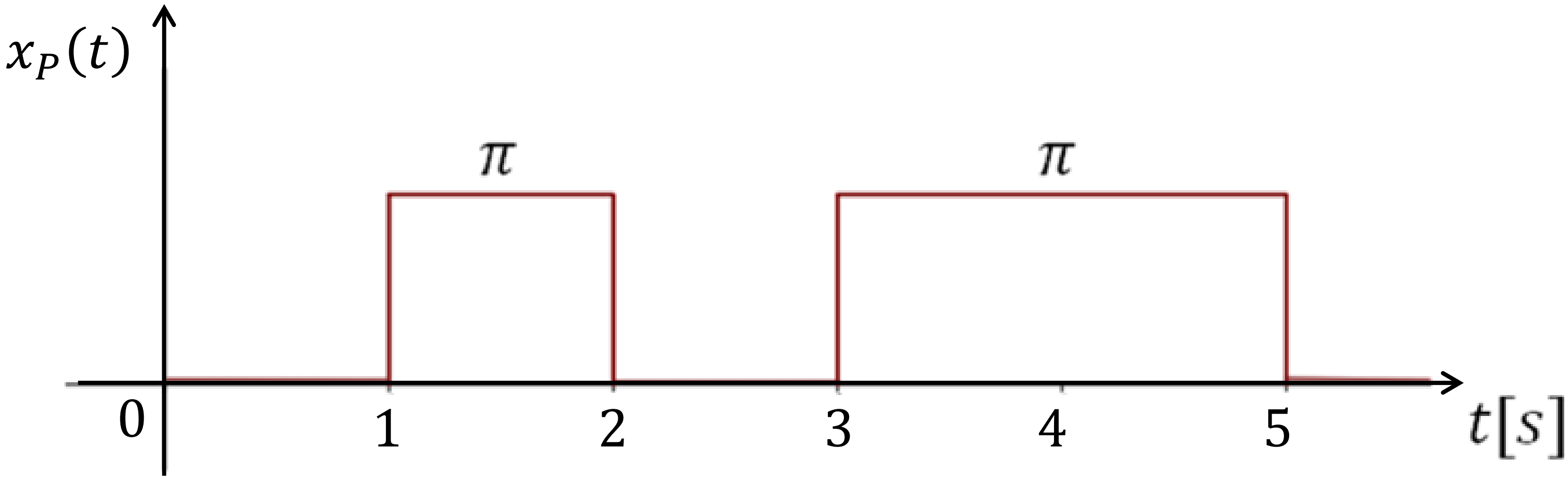
→ $x_A(t)$ is non-negative and modulates the amplitude of the carrier (see previous example)

→ $x_P(t)$ modulates the phase of the carrier

- $x_A(t)$ and $x_P(t)$ are real and baseband signals that encode information
- $x_c(t)$ is the passband modulated signal

Example: Phase Modulation

$x_A(t) = 1$

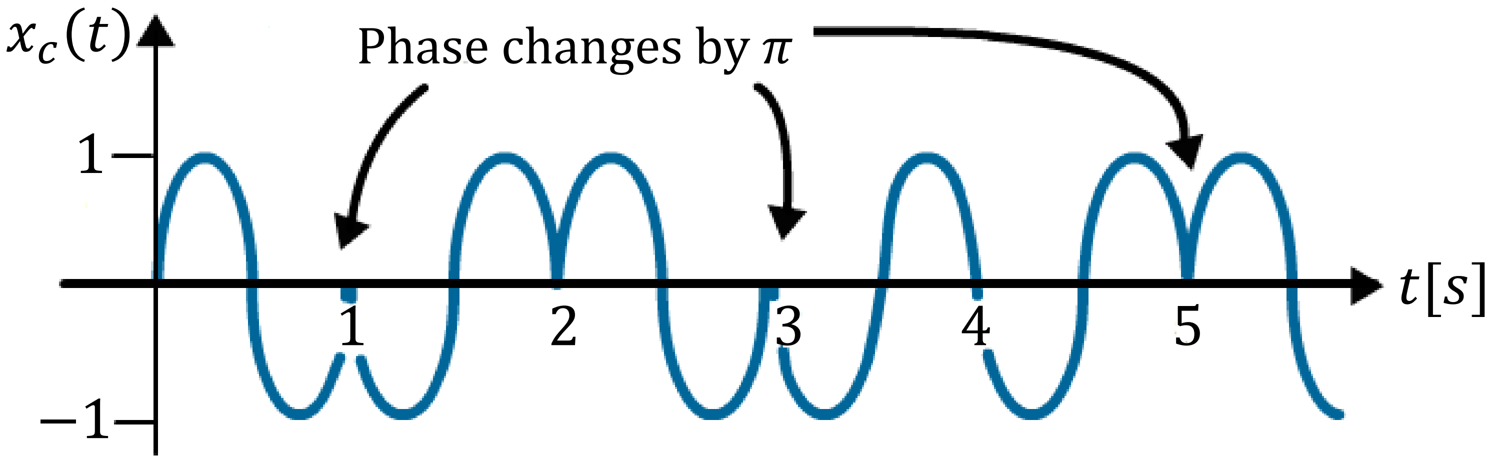


($x_P(t)$ encodes information 010110…)

Example: Phase Modulation

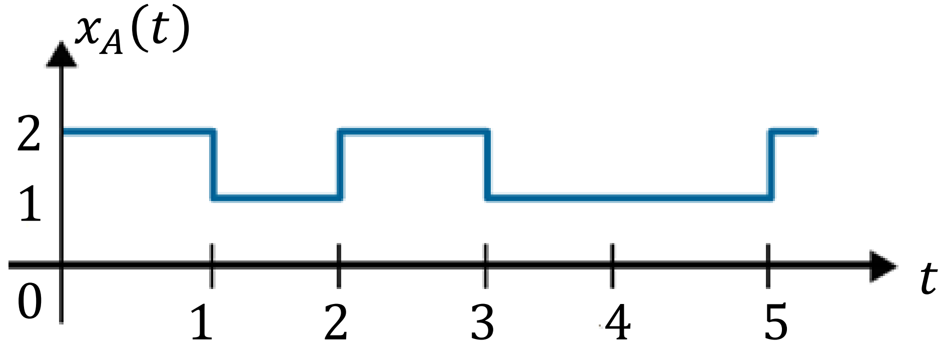
Modulated signal

$x_c(t) = \cos(2\pi f_c t + x_P(t))$ with $f_c = 1Hz$

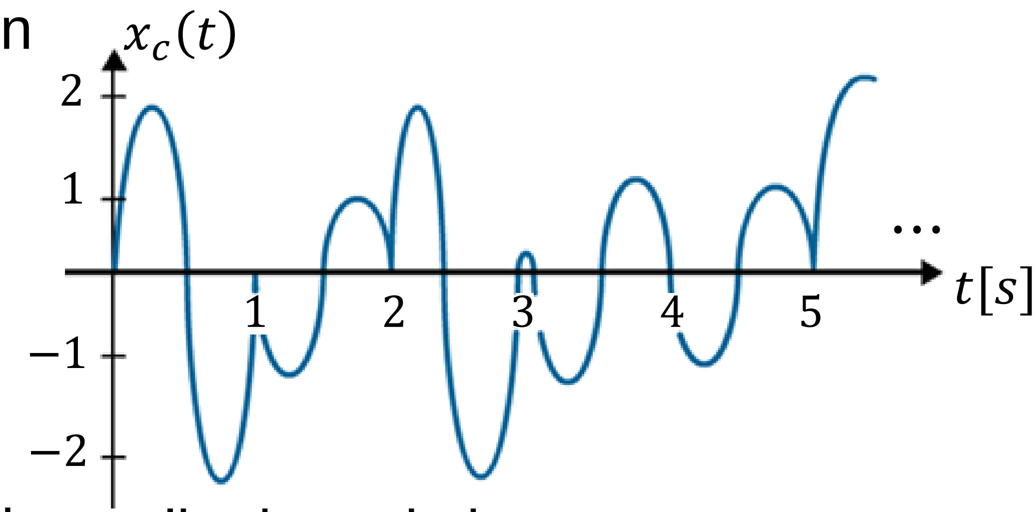


Example: Amplitude and Phase Modulation

- Ex.: Continuing the example from the previous slide assume that



($x_A(t)$ encodes information 101001…)



Remark: We can have both amplitude and phase modulation at the same time.

Complex Baseband Representation of Bandpass Signals

- Alternative form of the bandpass signal

$$x_c(t) = \sqrt{2}\, x_I(t) \cos(2\pi f_c t)$$
$$-\sqrt{2}\, x_Q(t) \sin(2\pi f_c t)$$

in-phase, or I, component

quadrature, or Q, component

⟶ Information is encoded by the two baseband signals $x_I(t)$ and $x_Q(t)$

Complex Baseband Representation of Bandpass Signals

→ $x_I(t)$ modulates the in-phase carrier $\cos(2\pi f_c t)$

→ $x_Q(t)$ modulates the quadrature carrier $-\sin(2\pi f_c t)$

- $x_I(t)$ and $x_Q(t)$ are real and baseband signals that encode information
- $x_c(t)$ is the passband modulated signal

Remark

- Amplitude modulation can also be thought of as in-phase modulation (with $x_I(t) = x_A(t)$ and $x_Q(t) = 0$).

Complex Baseband Representation of Bandpass Signals

- From amplitude/phase representation to in-phase quadrature representation:
  - Recall that $\cos(a+b) = \cos a \cos b - \sin a \sin b$
  - Using this formula, we obtain

$$
\begin{aligned}
x_c(t) &= \sqrt{2}\, x_A(t)\cos(2\pi f_c t + x_P(t)) \\
&= \sqrt{2}\, \underbrace{x_A(t)\cos x_P(t)}_{x_I(t)}\cos(2\pi f_c t) \\
&\qquad - \sqrt{2}\, \underbrace{x_A(t)\sin x_P(t)}_{x_Q(t)}\sin(2\pi f_c t)
\end{aligned}
$$

$$
\rightarrow \begin{cases} x_I(t) = x_A(t)\cos x_P(t) \\ x_Q(t) = x_A(t)\sin x_P(t) \end{cases}
$$

Complex Baseband Representation of Bandpass Signals

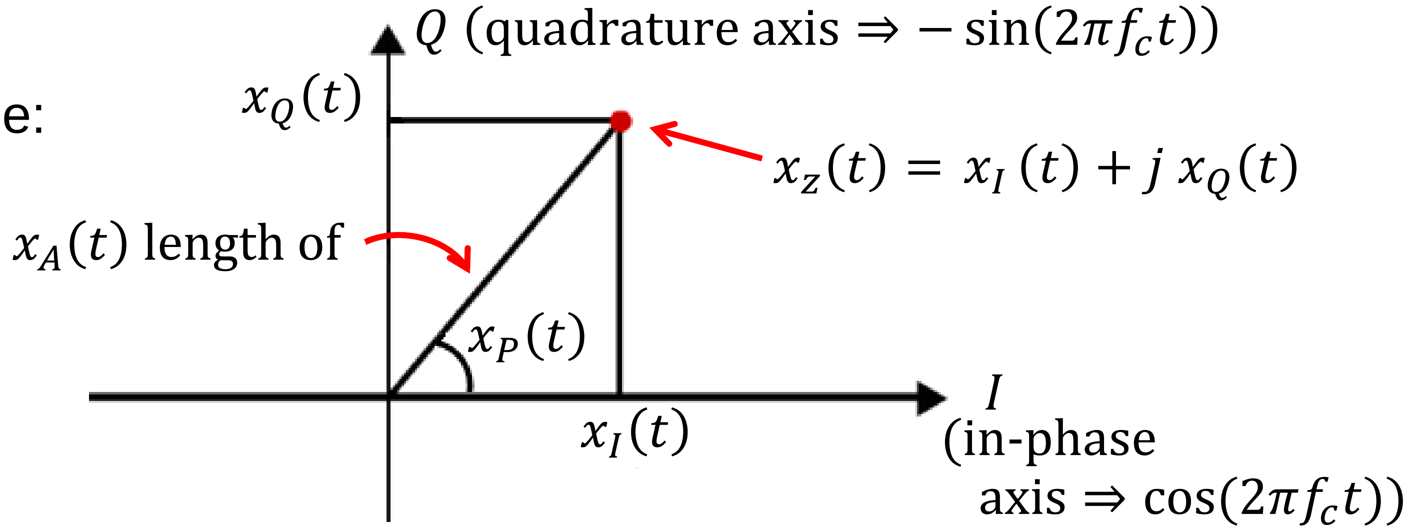
- From in-phase / quadrature representation to amplitude / phase representation:

  – From the equations on the previous slide, we get

$$\begin{cases} x_A(t) = \sqrt{x_I(t)^2 + x_Q(t)^2} \\ x_P(t) = \arg(x_I(t) + \mathrm{j}x_Q(\mathrm{t})) \end{cases}$$

Illustration

I-Q plane:



Information signal described by

$\left(x_I(t),\, x_Q(t)\right)$ ... cartesian coordinates

$\left(x_A(t),\, x_P(t)\right)$ ... polar coordinates

Remark: As t increases, the point • moves on the I-Q plane

Remark: $\cos\left(2\pi f_c t + \frac{\pi}{2}\right) = -\sin(2\pi f_c t)$

Complex Baseband Representation of Bandpass Signals

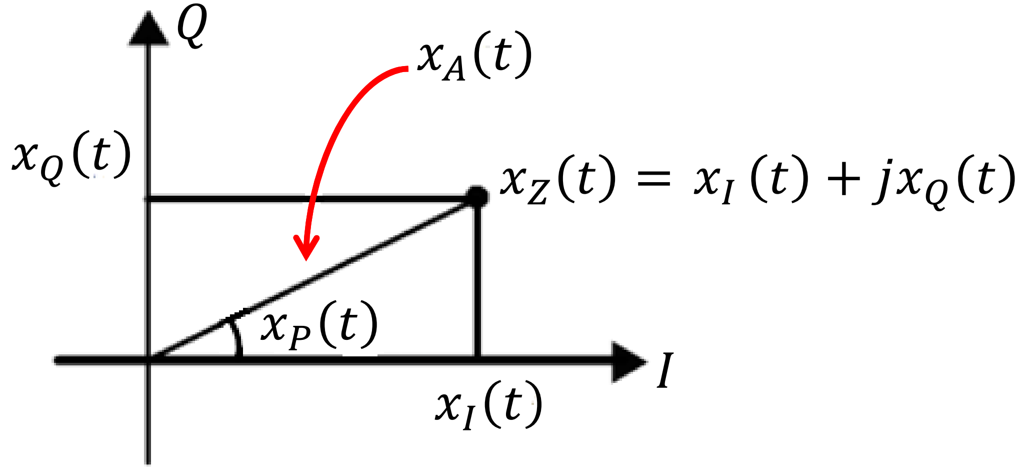
- Based on the discussion from the previous slide, the information signal is completely described by the pair of baseband signals $(x_A(t), x_P(t))$ or $\left(x_I(t), x_Q(t)\right)$.

- Complex baseband representation (or complex envelope) of the passband signal $x_c(t)$

$$x_z(t) = x_I(t) + j\, x_Q(t) = x_A(t)e^{j x_P(t)}$$

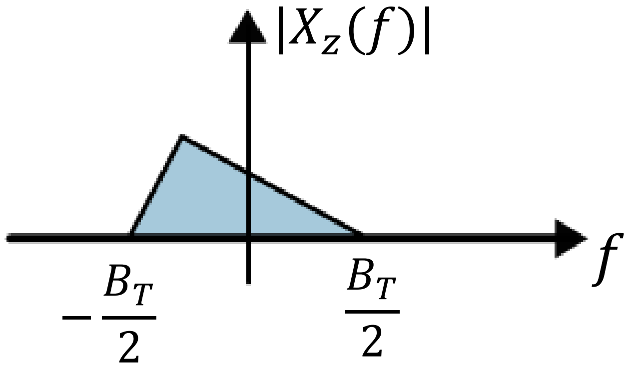
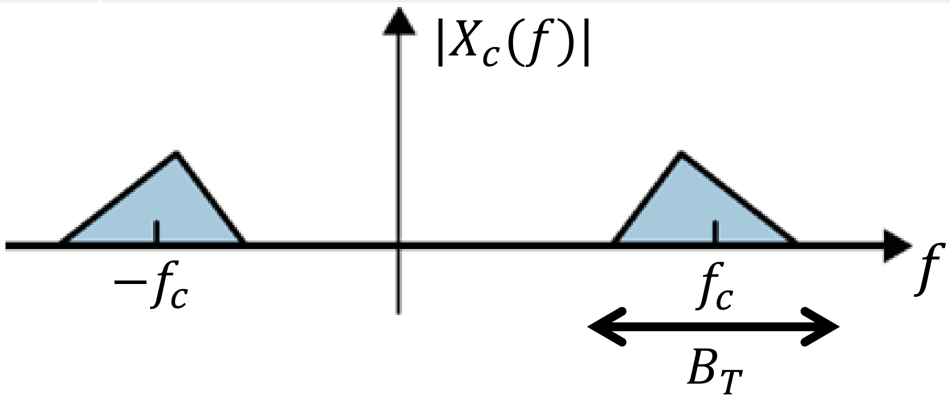
Complex Baseband Representation of Bandpass Signals

- Complex baseband representation



- $x_z(t)$ is complex and baseband

Summary

Information signal	**Modulated signal**
- Complex envelope $x_z(t)$	- Passband signal $x_c(t)$
- Complex and baseband	- Real and passband
 (no symmetry in general)	 (Hermitian symmetry)

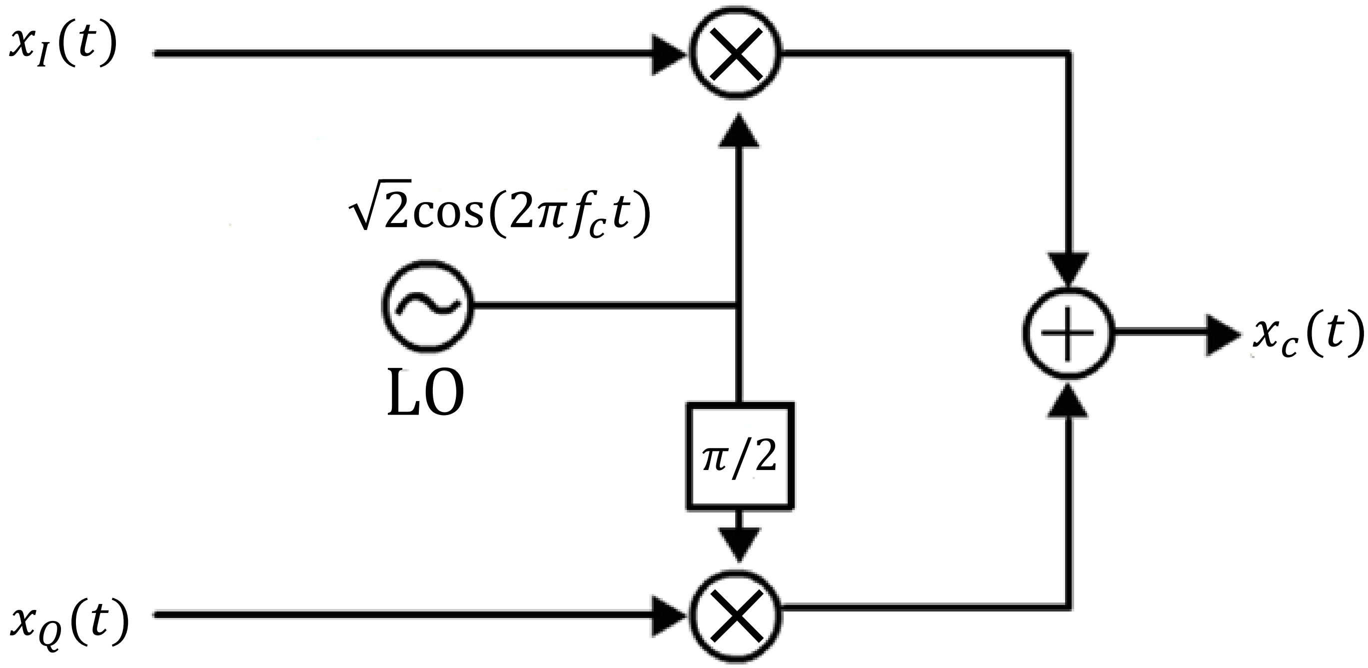
Summary

Information signal	Modulated signal
- $x_z(t) = x_I(t) + j\, x_Q(t)$ $= x_A(t) e^{j x_P(t)}$	- $x_c(t) = \sqrt{2}\, x_A(t) \cos(2\pi f_c t + x_P(t))$ $= \sqrt{2}\, x_I(t) \cos(2\pi f_c t)$ $-\sqrt{2}\, x_Q(t) \sin(2\pi f_c t)$ $= \sqrt{2}\, Re\left\{x_z(t) e^{2\pi f_c t}\right\}$

- Ex.: See Ex. 4.1

Upconversion

- From complex envelope to bandpass signal



LO: Local Oscillator

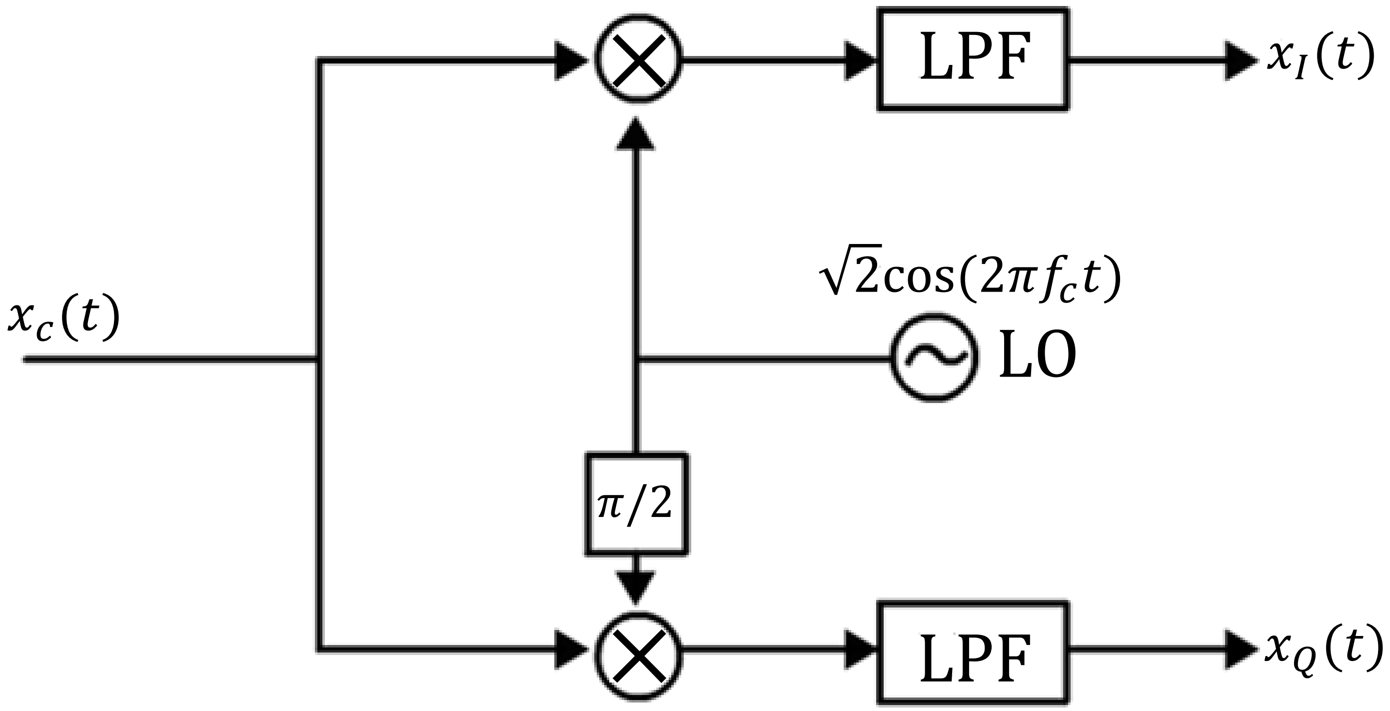
Upconversion

Remarks:

- Upconversion is performed by the transmitter to modulate the information signal $x_z(t)$
- The complex envelope $x_z(t)$ is often processed digitally (software defined radio, SDR)

Downconversion

- From bandpass signal to complex envelope



LO: Local Oscillator
LPF: Low-Pass Filter

Remarks

- Downconversion is performed at the receiver
- The downconverted baseband complex envelope is often processed digitally (SDR)

Remarks

- Why do we need LPF?

$$x_c(t)\sqrt{2}\cos(2\pi f_c t) = 2x_P(t)\,(\cos(2\pi f_c t))^2$$

$$-\sqrt{2}\;x_Q(t)\sin(2\pi f_c t)\cos(2\pi f_c t)$$

from LO

$$= x_I\;(t) + \boxed{x_I(t)\cos(4\pi f_c t) - x_Q(t)\sin(4\pi f_c t)}$$

$$\cos a \cos b = 1/2\;(\cos(a-b) - \cos(a+b))$$

$$\cos a \sin b = 1/2\;(\sin(a+b) - \sin(a-b))$$

Removed by LPF

And similarly for $x_c(t)\left(-\sqrt{2}\sin(2\pi f_c t)\right) =$

$$= x_Q(t)\boxed{- x_Q(t)\cos(4\pi f_c t) - x_I(t)\cos(4\pi f_c t)}$$

Remarks

- See Sec. 4.3 for discussion on visualization.

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