

NJIT



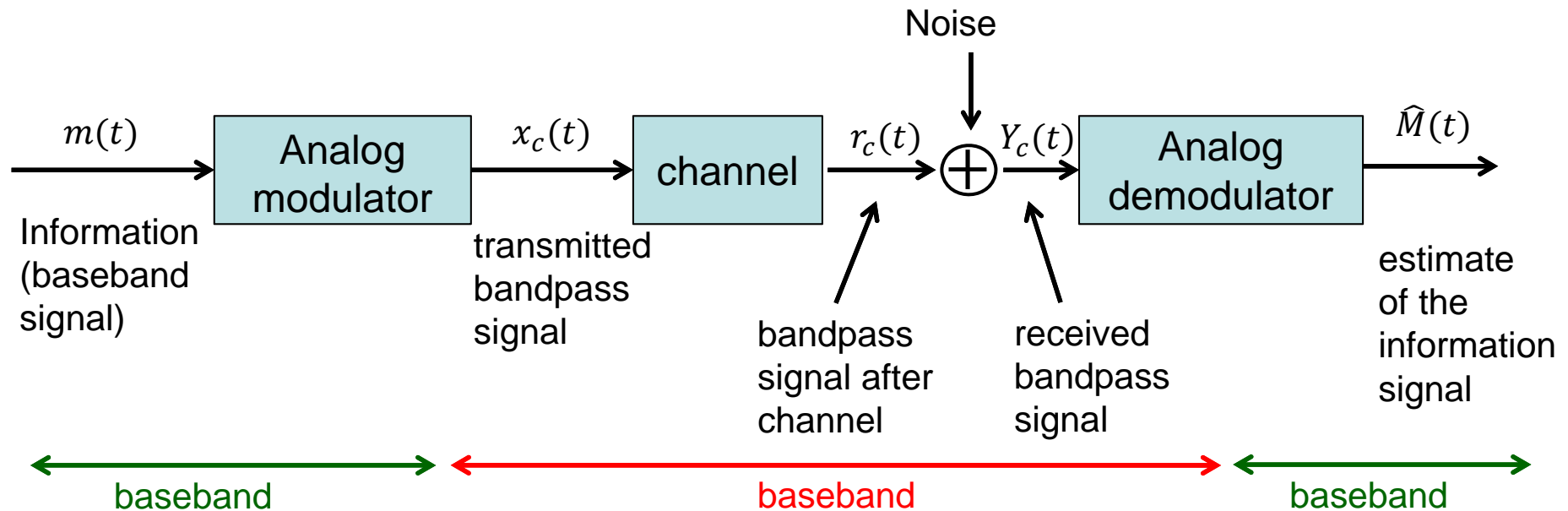
New Jersey's Science &
Technology University

THE EDGE IN KNOWLEDGE

Analog Communication

Basics (Chapter 5)

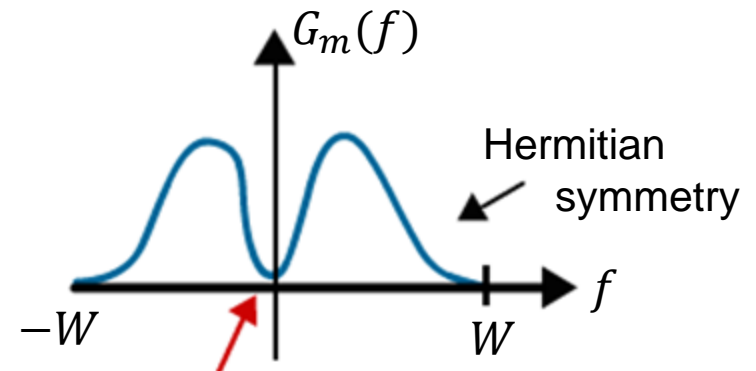
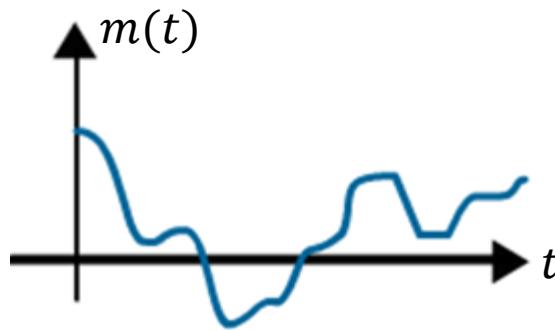
- Block diagram:



Remark: We use capital letters to denote random quantities (e.g., dependent on noise) – to be discussed

Analog Communication

- The information message $m(t)$ is real and encodes analog information (ex: audio, video)



DC often zero (corresponds to average value, which carries no information)

Ex.: $W = 3.5 - 4 \text{ KHz}$ human voice

$W = 15 - 20 \text{ KHz}$ high-fidelity audio

$W = 4.5 \text{ MHz}$ (raster scan) NTSC TV

See also Ex. 5.1 – 5.3

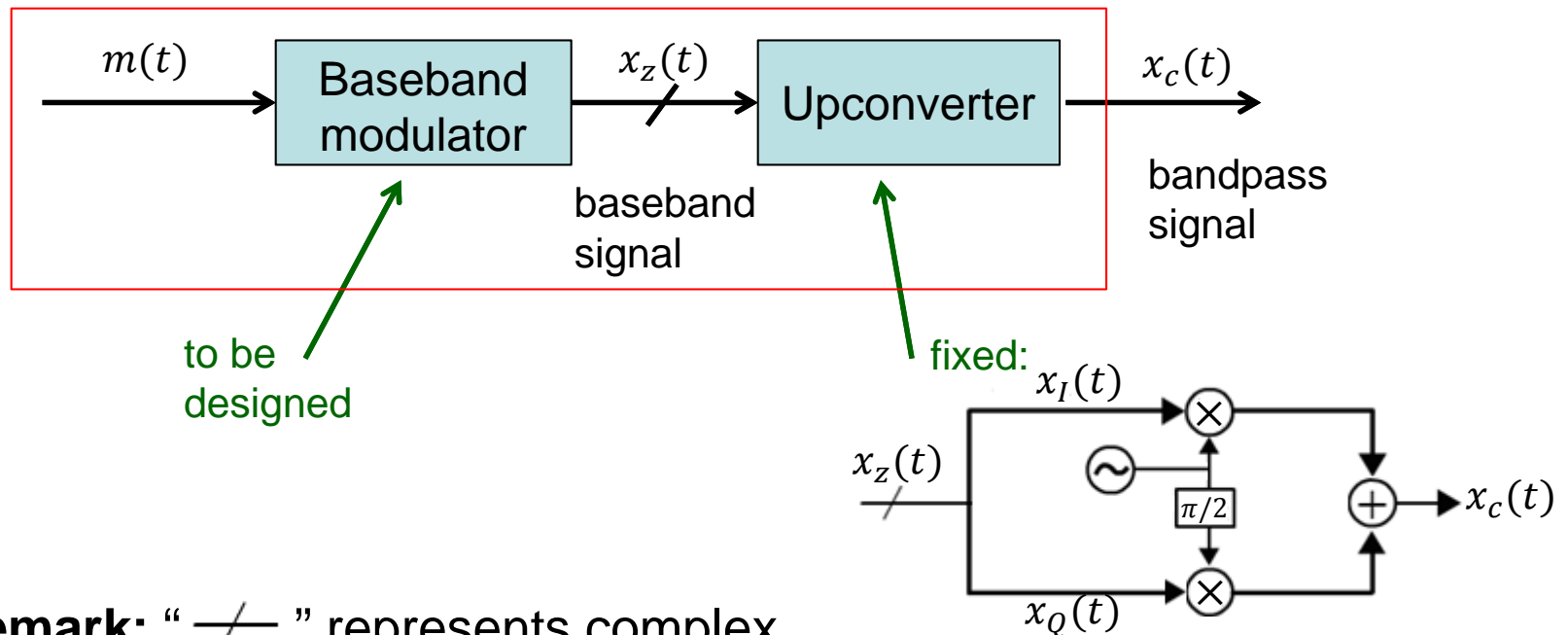


Analog Communication

- Goal: Design modulator and demodulator so that $\hat{m}(t)$ is “very close” to $m(t)$ despite channel and noise
- Constraints: transmission power and bandwidth, message bandwidth

Analog Communication

- Modulator:



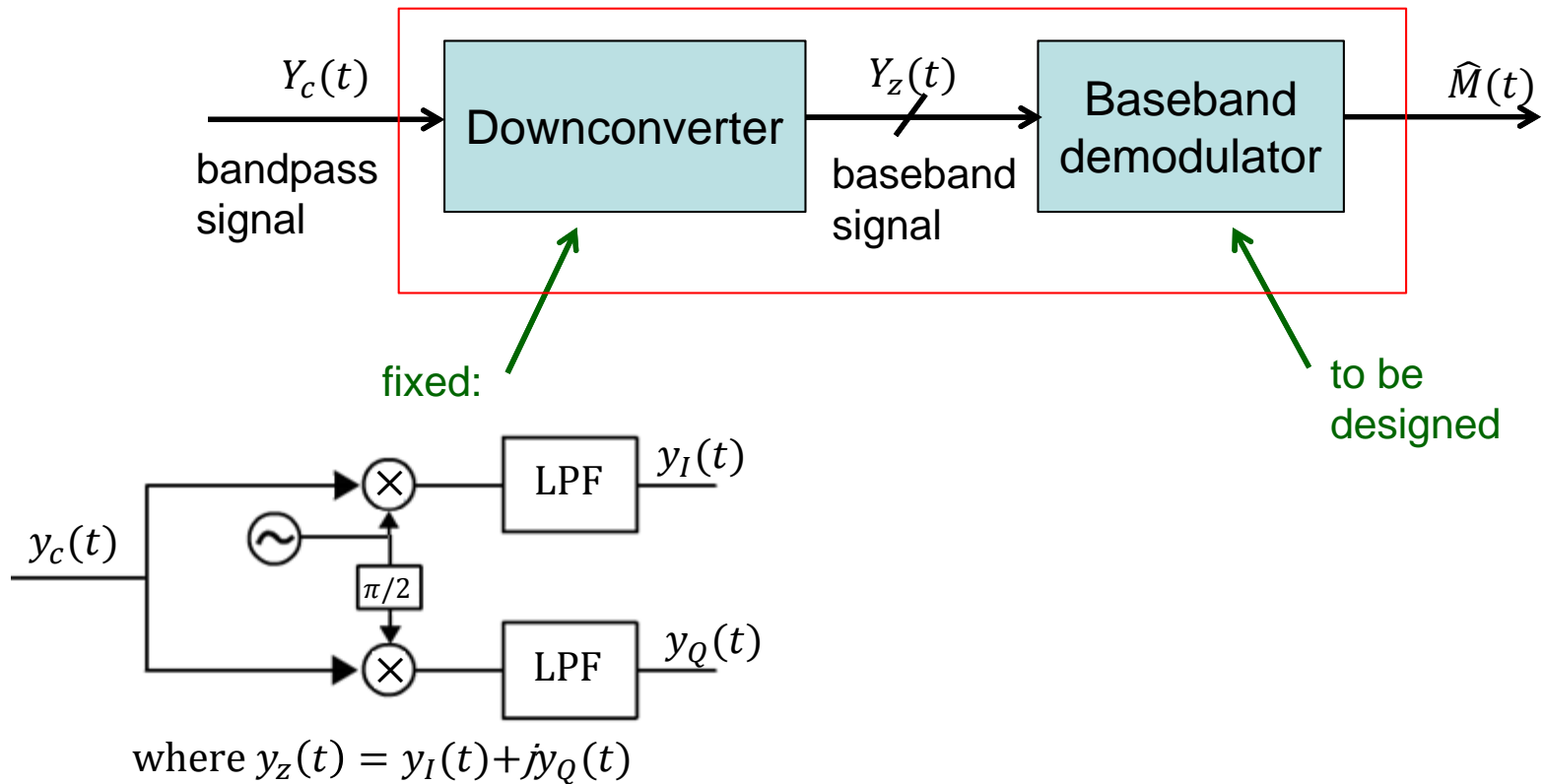
Remark: “ $\text{—}/\text{—}$ ” represents complex signals (i.e., two real signals)

Analog Communication

Modulation scheme	Information encoded in
Amplitude Modulation (AM)	$x_I(t)$ or $x_A(t)$
Phase Modulation (PM)	$x_P(t)$
Frequency Modulation (FM)	$\frac{d}{dt}x_P(t)$ (instantaneous frequency)

Analog Communication

- Demodulator:



Analog Communication

- Channel:
 - Represents the effect of the propagation between transmitter and receiver
 - Simple model:

$$r_c(t) = \sqrt{L_P} X_c(t - \tau_p)$$

Power attenuation, or path loss:
typically measured in *dB* as

$$L_p[dB] = -10 \log_{10} L_p$$

(e.g., an attenuation of *10dB*
corresponds to $L_p = 10^{-\frac{10}{10}} = 0.1$)

propagation delay

$$\tau_p = \frac{\text{distance } tx - rx}{c}$$

(e.g., if distance = 1Km

$$\tau_p = \frac{10^3}{3 \cdot 10^8} = \frac{1}{3} 10^{-5} s \\ \approx 3.3 \mu s)$$

Analog Communication

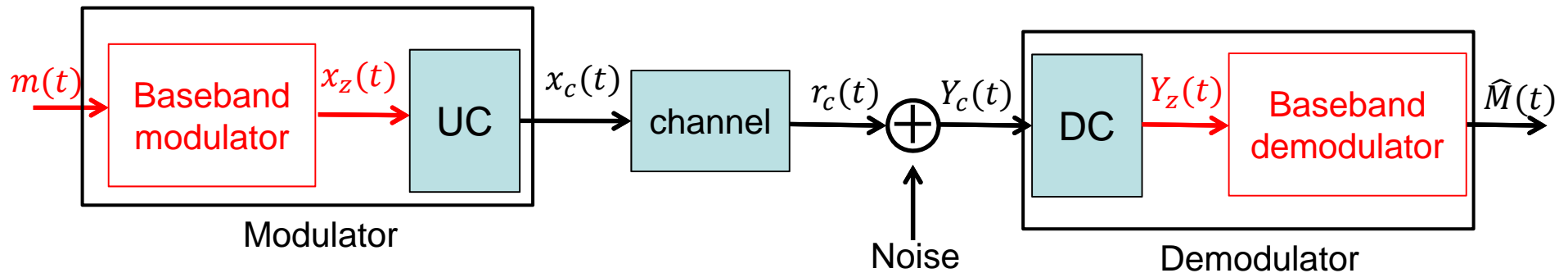
- Given the simple model above, we get

$$\begin{aligned}r_c(t) &= \sqrt{L_P} x_c(t - \tau_p) \\&= \sqrt{L_P} \sqrt{2} \operatorname{Re}\{x_z(t - \tau_p) e^{j2\pi f_c(t - \tau_p)}\} \\&= \sqrt{2} \operatorname{Re}\left\{ \underbrace{\sqrt{L_P} x_z(t - \tau_p) e^{-j2\pi f_c \tau_p}}_{r_z(t): \text{ complex envelope of } r_c(t)} e^{j2\pi f_c t} \right\} \\&= \sqrt{2} \operatorname{Re}\{r_z(t) e^{j2\pi f_c t}\} \\&\text{with } r_z(t) = \sqrt{L_P} x_z(t - \tau_p) e^{j\phi_p} \quad (\phi_p = -2\pi f_c \tau_p)\end{aligned}$$

- We will mostly set $L_p = 1$ and $\tau_p = 0$ (but $\phi_p \neq 0$)
- Remark:** A receiver that knows ϕ_p is called coherent, and is otherwise called non-coherent.

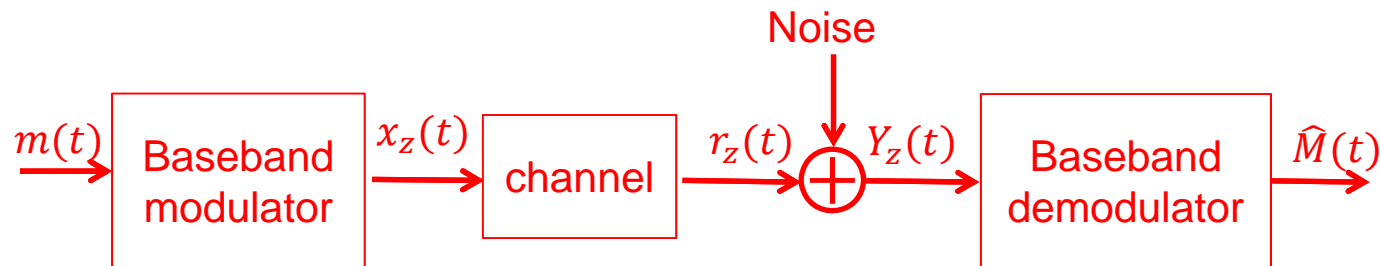
Analog Communication

- We can now represent the analog communication system in terms of baseband signals only:
 - Original system (UC - upconverter, DC - downconverter):



Analog Communication

- We can now represent the analog communication system in terms of baseband signals only:
 - Equivalent baseband system:



Remark: As we will see, in the original system, the noise is bandpass, while it is baseband in the equivalent baseband system.

Performance Metrics

- Complexity: affects cost of implementation
- Fidelity:
 - Measures how well $\hat{M}(t)$ represents $m(t)$
 - Typically measured by signal-to-noise ratio on the message:

$$\hat{M}(t) = Am(t) + N(t)$$

estimate of the information signal information signal Noise (estimation error)

$$SNR_m = \frac{A^2 P_m}{P_N}$$

SNR for message demodulation Power of the info signal Power of the noise:
 $P_N = E[N(t)^2]$

Expectation of a random variable (to be discussed)

```
graph TD
    Eq1["\hat{M}(t) = Am(t) + N(t)"]
    Eq2["SNR_m = (A^2 P_m) / P_N"]
    Eq3["P_N = E[N(t)^2]"]
    
    Eq1 -- "estimate of the information signal" --> M_hat["\hat{M}(t)"]
    Eq1 -- "information signal" --> m_t["m(t)"]
    Eq1 -- "Noise (estimation error)" --> N_t["N(t)"]
    
    Eq2 -- "SNR for message demodulation" --> SNR_m["SNR_m"]
    Eq2 -- "Power of the info signal" --> P_m["P_m"]
    Eq2 -- "Power of the noise:" --> P_N["P_N"]
    
    Eq3 -- "Expectation of a random variable (to be discussed)" --> E["E["]
```

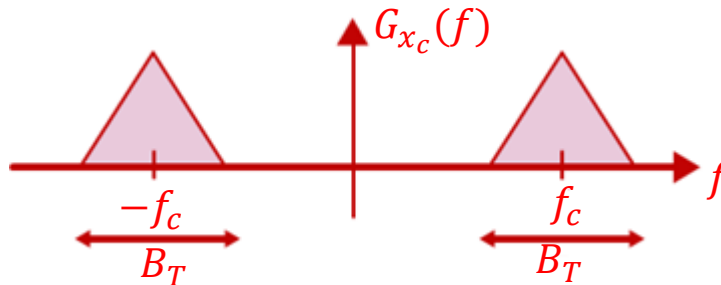
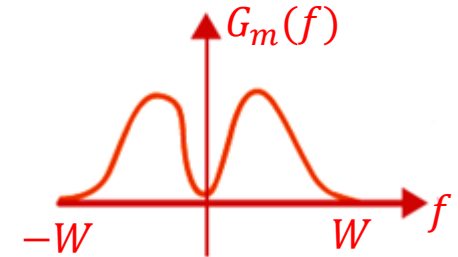
Performance Metrics

- Spectral efficiency:
 - Measures how well the system uses the available bandwidth resources:

$$E_B = \frac{W}{B_T}$$

bandwidth of the information signal

transmission bandwidth



Performance Metrics

Ex.:

System	B_T	W	E_B
AM radio	8 kHz	4 kHz	50%
FM radio	180 kHz	15 kHz	8.3%
TV	6M Hz	4.5 MHz	75%



Performance Metrics

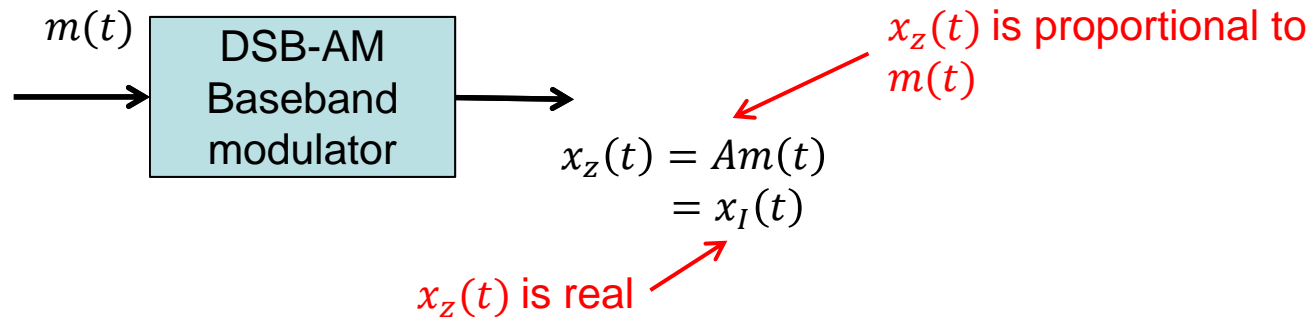
- Transmission SNR:

$$SNR_t = \frac{\text{power of the received signal } x_c(t) \text{ (or } x_z(t))}{\text{power of the received noise}}$$

→ Performance defined by (E_B, SNR_m, SNR_t)

Amplitude Modulation (AM)

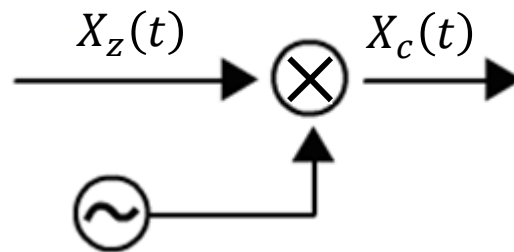
- We focus here on the simplest AM scheme, namely Double Side Band AM (DSB-AM)
- Baseband modulator of DSB-AM:



- Remark: If $m(t) \geq 0 \Rightarrow x_z(t) = x_A(t)$, from which the name AM

Amplitude Modulation (AM)

- Since $x_z(t)$ is real, the upconverter following the baseband modulator simplifies as



- The resulting bandpass signal $x_c(t)$ is given by

$$x_c(t) = \sqrt{2}x_z(t)\cos(2\pi f_c t) = \sqrt{2}Am(t)\cos(2\pi f_c t)$$

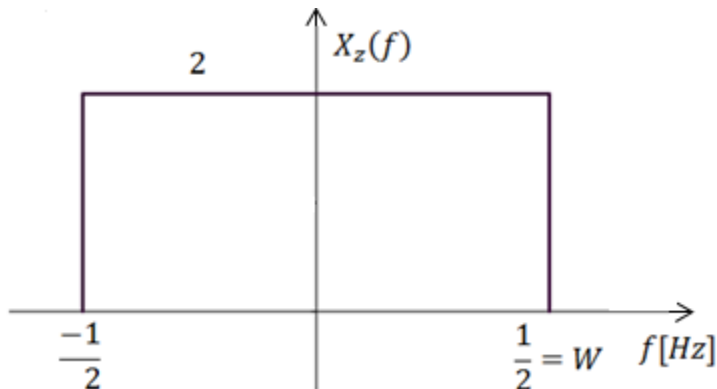
and

$$X_c(f) = \frac{A}{\sqrt{2}} M(f - f_c) + \frac{A}{\sqrt{2}} M(f + f_c)$$

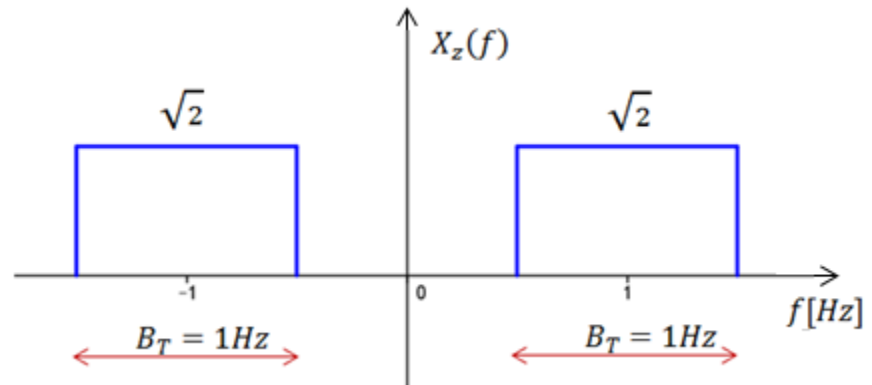
Amplitude Modulation (AM)

Ex.: Information signal: $m(t) = \text{sinc}(t)$; $A = 2$; $f_c = 1\text{Hz}$

$$x_z(t) = 2\text{sinc}(t)$$



$$x_c(t) = 2\sqrt{2}\text{sinc}(t) \cos(2\pi t)$$

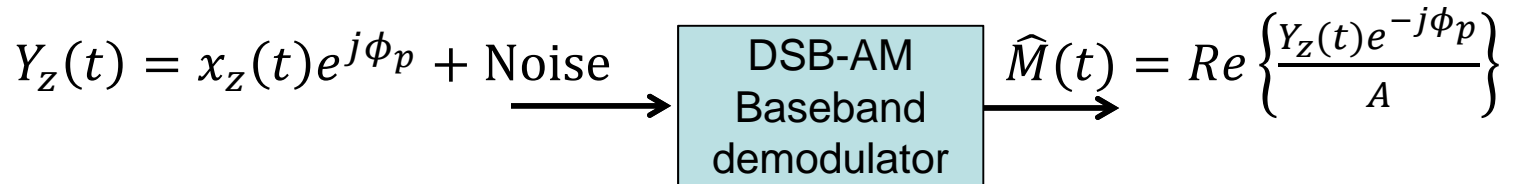


Try to plot $x_z(t)$ and $x_c(t)$ (in time domain)



Amplitude Modulation (AM)

- Baseband demodulator of DSB-AM:



- We have:

$$\begin{aligned} \hat{M}(t) &= \text{Re} \left\{ \left(\frac{x_Z(t)e^{j\phi_p} + \text{Noise}}{A} \right) e^{-j\phi_p} \right\} \\ &= \text{Re} \left\{ \frac{x_Z(t)}{A} \right\} + \text{Re} \left\{ \frac{\text{Noise}}{A} e^{-j\phi_p} \right\} \end{aligned}$$

$$x_Z(t) = Am(t) \text{ is real} \longrightarrow = m(t) + \text{Noise}$$

Amplitude Modulation (AM)

So, apart from the noise, we have recovered $m(t)$

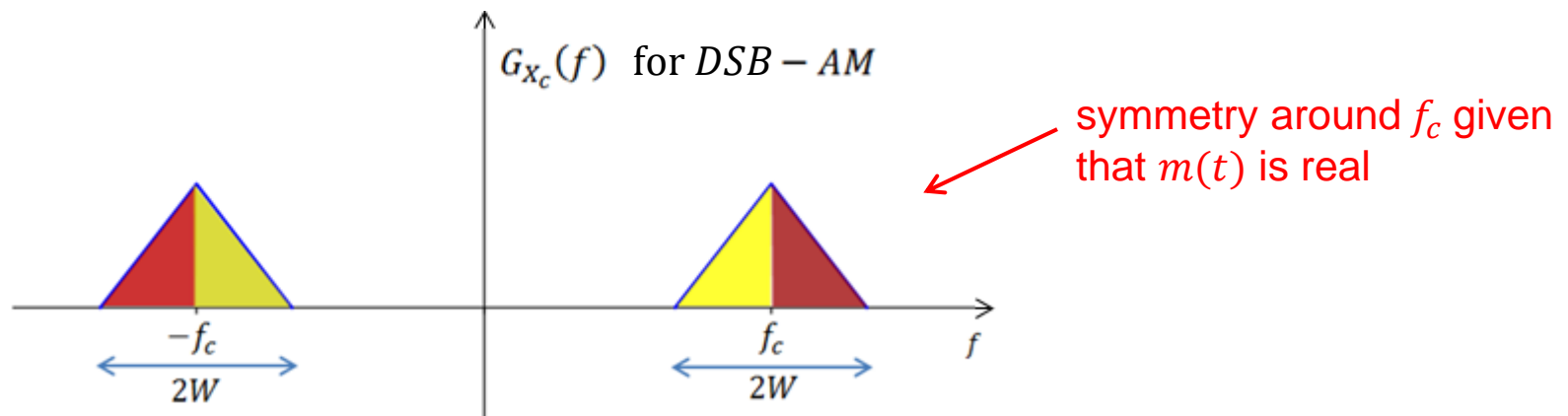
- Remark: This demodulation strategy assumes knowledge of ϕ_p and is hence coherent (see textbook on how to estimate ϕ_p from $(Y_z(t))^2$ via $\frac{1}{2} \arg((Y_z(t))^2)$).

Amplitude Modulation (AM)

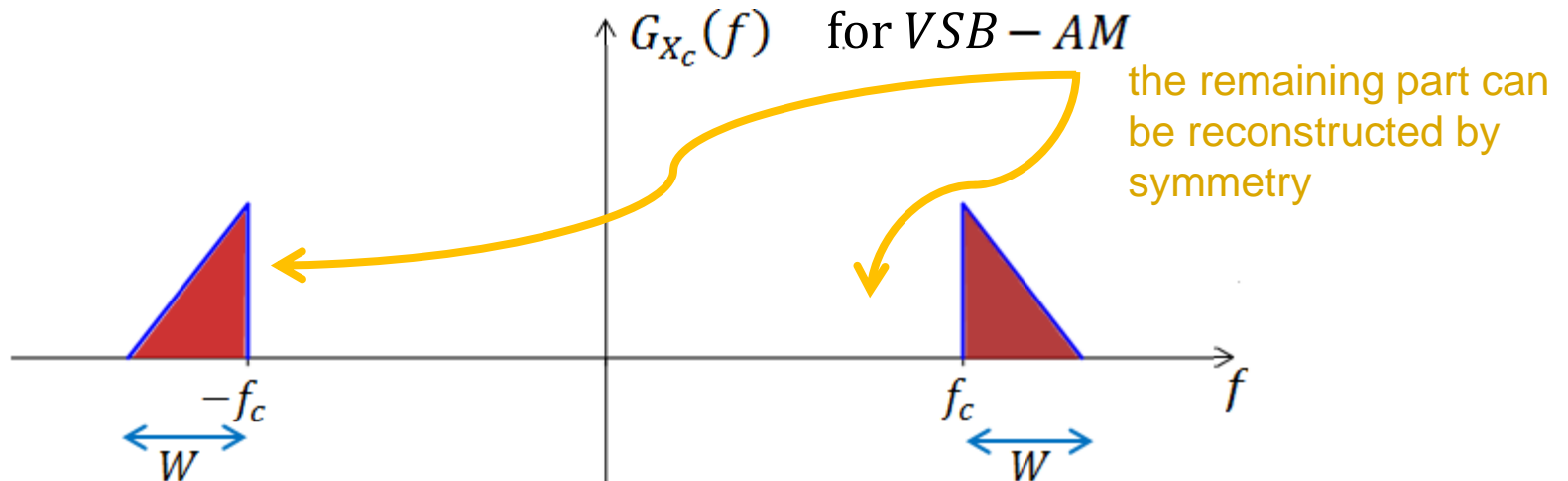
- Spectral efficiency of DSB-AM:

$$\text{since } B_T = 2W, \text{ we have } E_B = \frac{W}{B_T} = 0.5 \text{ (50\%)}$$

- This can be improved by a method called Vestigial Sideband AM (VSB-AM), used, e.g., for (analog) TV transmission:



Amplitude Modulation (AM)



⇒ Spectral efficiency of VSB-AM:

$$E_B = \frac{W}{W} = 100\%$$

Amplitude Modulation (AM)

- Remark: The baseband complex envelope $x_z(t)$ of a VSB-AM signal is complex, unlike DSB-AM. Therefore, VSB-AM uses both the $\cos(2\pi f_c t)$ and the $\sin(2\pi f_c t)$ carriers, which doubles the spectral efficiency (see also pages 6.16-6.27)
- Additional (optional) reading: Large-Carrier AM on pages 6.9-6.16 – simpler decoding, but larger transmission power.

Angle (Phase and Frequency) Modulation

- While DSB-AM modulates $x_I(t)$ (or $x_A(t)$), angle modulation modulates $x_P(t)$
- Baseband modulator: $x_z(t) = Ae^{jx_P(t)}$
 - Phase modulation (PM): $x_P(t) = K_p m(t)$
 - Frequency modulation (FM):

$$x_p(t) = x_P(t_0) + K_f \int_{t_0}^t m(\lambda) d\lambda$$

Angle (Phase and Frequency) Modulation

- FM modulates the instantaneous frequency

$$\underbrace{\frac{1}{2\pi} \frac{dx_p(t)}{dt}}_{\text{instantaneous frequency}} = \frac{K_f}{2\pi} m(t)$$

- See Fig. 7.1 – 7.3 for illustrations

Angle (Phase and Frequency) Modulation

Ex.: $m(t) = \cos(2\pi t)$; $A = 1$; $K_p = 1$; $K_f = 1$

$$\text{PM: } x_p(t) = \cos(2\pi t)$$

$$x_c(t) = \sqrt{2} \cos(2\pi f_c t + \cos(2\pi t))$$

$$\text{FM: } x_p(t) = \int_{-\infty}^t \cos(2\pi \lambda) d\lambda = \frac{1}{2\pi} \sin(2\pi t)$$

$$x_c(t) = \sqrt{2} \cos(2\pi f_c t + \frac{1}{2\pi} \sin(2\pi t))$$



Angle (Phase and Frequency) Modulation

- As you can easily hear comparing AM and FM radios, FM has a much better fidelity. How is this accomplished?
- Basic idea: Angle modulation trades a larger bandwidth, and hence a lower spectral efficiency, for a better fidelity:

$\uparrow B_T \longrightarrow \uparrow \text{fidelity}$

Angle (Phase and Frequency) Modulation

- Calculation of B_T for angle modulation is not easy (see pages 7.7 – 7.16)

→ Carson's rule

$$B_T \approx 2(D + 1)W = (D + 1) \times \text{bandwidth for DSB-AM}$$

↑
approximation

where $D = (\text{largest instantaneous frequency})/W$

$$= \frac{1}{2\pi W} \max_t \left| \frac{dx_p(t)}{dt} \right|$$

Angle (Phase and Frequency) Modulation

- For PM:

$$D = \frac{K_p}{2\pi W} \max_t \left| \frac{dm(t)}{dt} \right|$$

- For FM:

$$D = \frac{K_f}{2\pi W} \max_t |m(t)|$$

- It can be shown (to be discussed) that

$\uparrow D \longrightarrow \uparrow \textit{fidelity}$

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