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New Jersey's Science &
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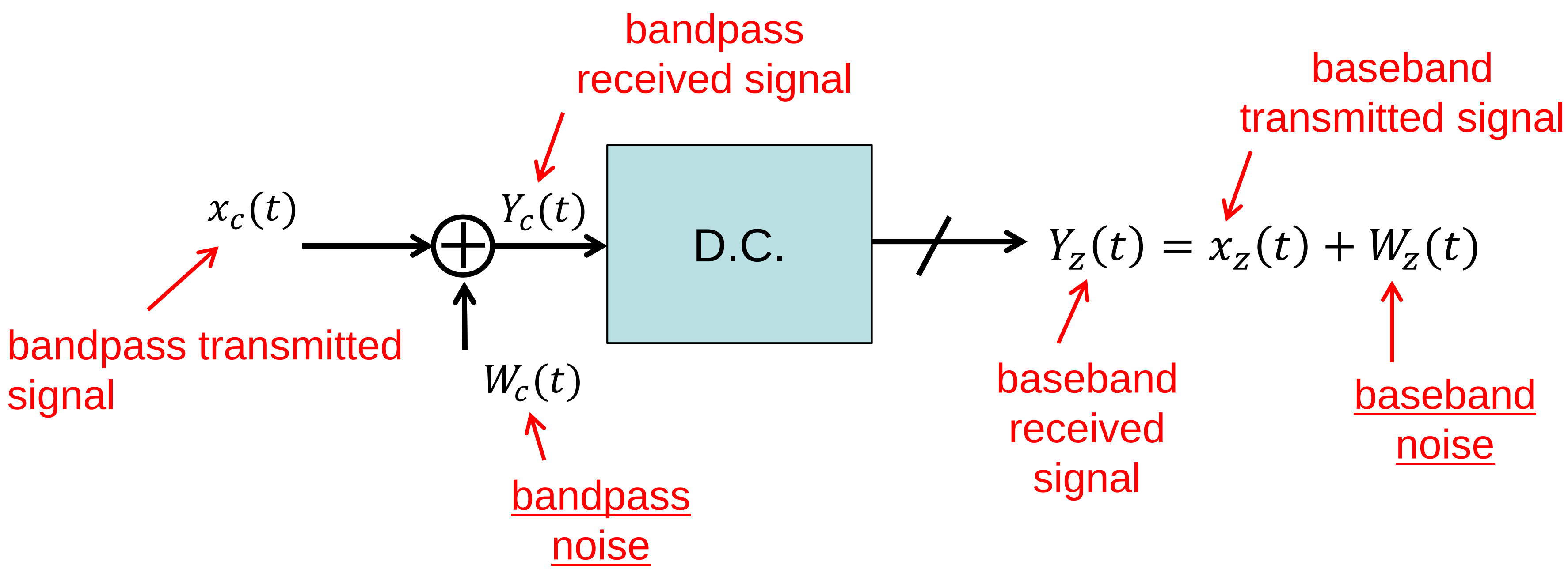
THE EDGE IN KNOWLEDGE

Noise in Bandpass Communication Systems

(Chapter 10)

- Receiver's side of a communication system (set $\phi_p = 0$ for simplicity)



(D.C.: Downconverter)

Noise in Bandpass Communication Systems

- Recall that $Y_z(t) = Y_I(t) + jY_Q(t)$:

$$Y_I(t) = x_I(t) + W_I(t)$$

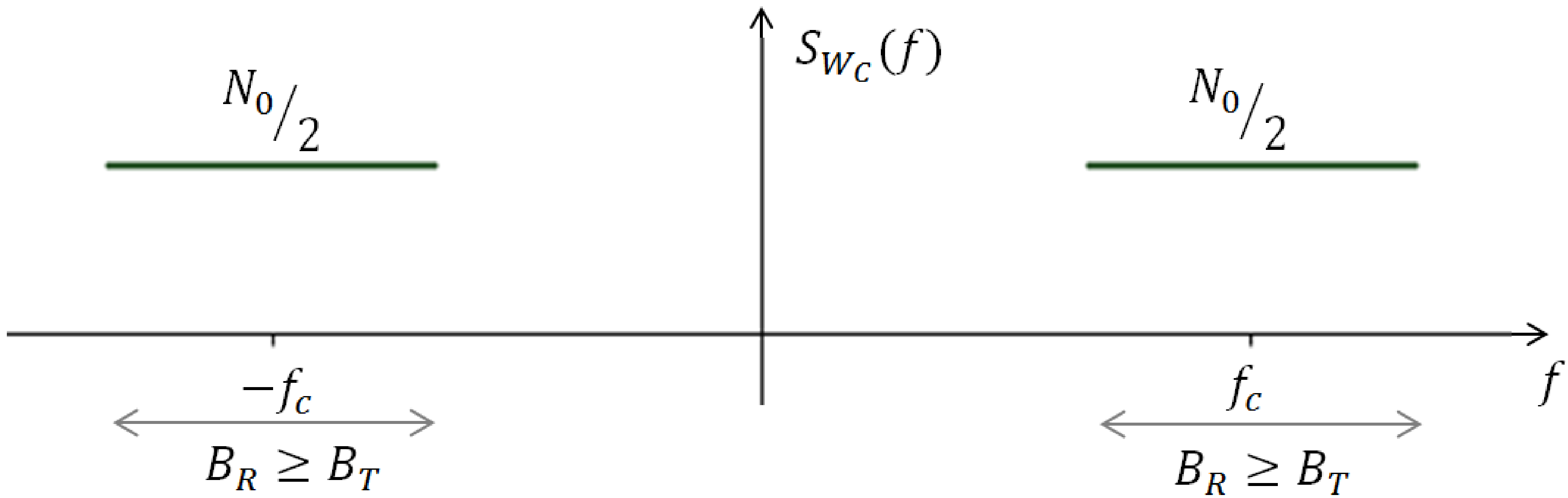
in-phase component of the baseband signal

$$Y_Q(t) = x_Q(t) + W_Q(t)$$

quadrature component of the baseband signal

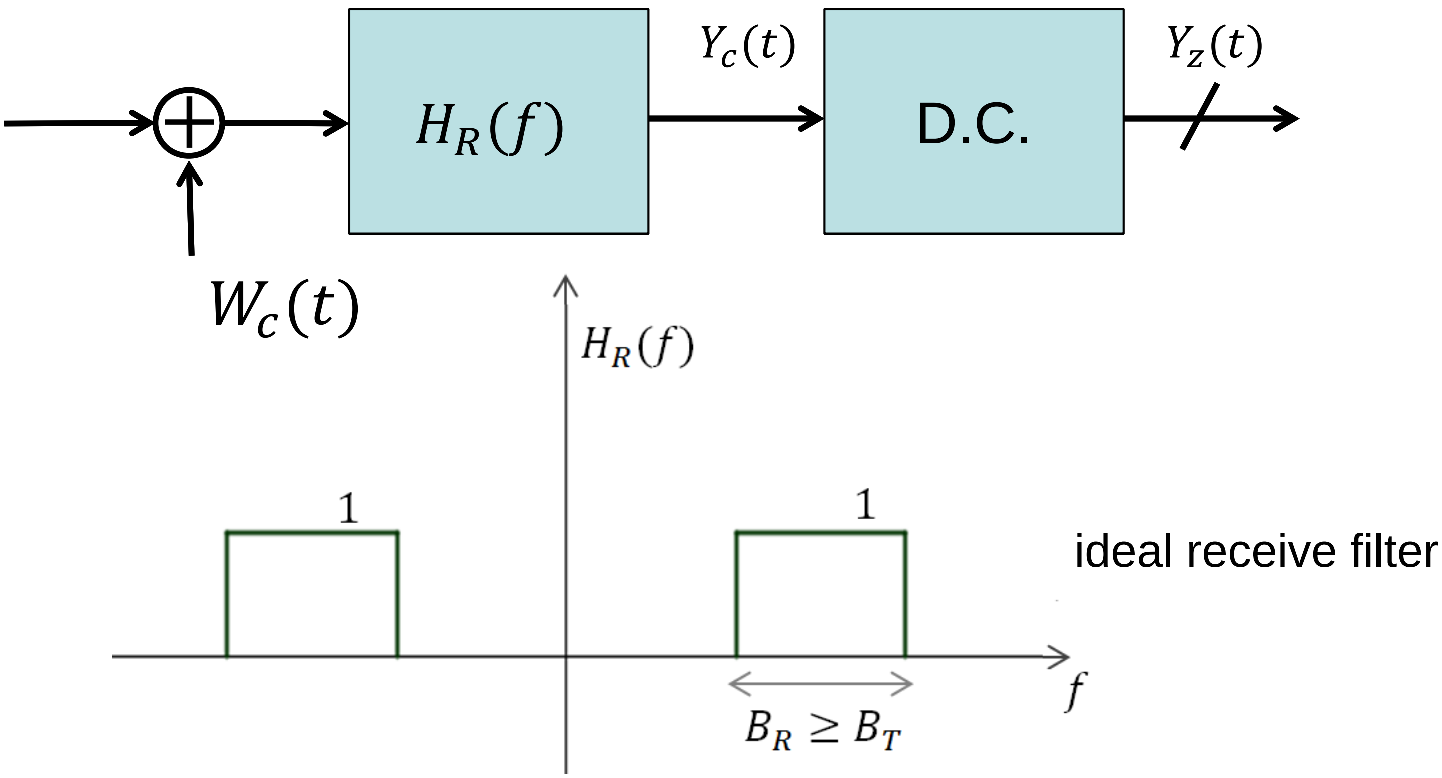
Noise in Bandpass Communication Systems

- The bandpass noise $W_c(t)$ is a WGN with power spectral density $S_{w_c}(f) = \frac{N_0}{2}$ within the bandwidth of the receiver.



Noise in Bandpass Communication Systems

- In practice, the receiver always filters the received signal $Y_c(t)$ before downconversion (not shown in diagram seen in the previous slide for simplicity):



Noise in Bandpass Communication Systems

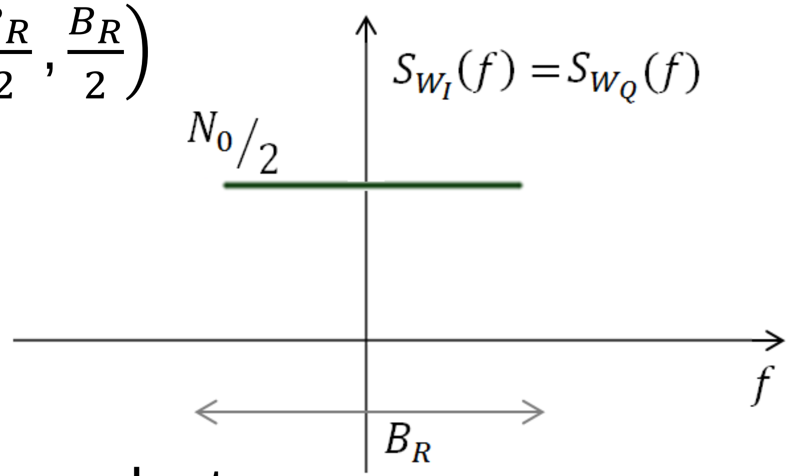
- What can we say about the baseband noise
  $W_z(t) = W_I(t) + jW_Q(t)$?

- It can be proved that

  a) $W_I(t)$ and $W_Q(t)$ are WGN with power spectral densities

$$S_{W_I}(f) = S_{W_Q}(f) = \frac{N_0}{2}$$

within the bandwidth $\left(-\frac{B_R}{2}, \frac{B_R}{2}\right)$



  b) $W_I(t), W_Q(t)$ are independent

Noise in Bandpass Communication Systems

Remark:

$$E[W_c(t)^2] = \frac{N_0}{2} \cdot 2B_R = N_0 B_R$$

$$\left.\begin{aligned} E[W_I(t)^2] &= \frac{N_0}{2} B_R \\ E[W_Q(t)^2] &= \frac{N_0}{2} B_R \end{aligned}\right\} \rightarrow \begin{aligned} &E[|W_z(t)|^2] = \\ &= E[W_I(t)^2] + E[W_Q(t)^2] \\ &= N_0 B_R \end{aligned}$$

$\longrightarrow$ The power of the bandpass noise, $E[W_c(t)^2]$ is equal to the power of the baseband signal, $E[|W_z(t)|^2]$
(as for energy for bandpass/baseband energy signals)

Noise in Bandpass Communication Systems

- Generating a baseband WGN $W_z(t)$ with power $P_{W_z}(= N_0 B_R)$ in MATLAB

$W_I = sqrt(P_{W_z}/2)^*$randn (N,1);

% generates N samples of $W_I(t)$

$W_Q = sqrt(P_{W_z}/2)^*$randn (N,1);

% generates N samples of $W_Q(t)$

$W_z = W_I + j * W_Q$;

% generates N samples of $W_z(t)$

Fidelity in Analog Communication Systems

(Chapter 11)

- As an application of the theory described in previous slides, we now evaluate the SNR for the considered analog modulations.

Fidelity in Analog Communication Systems

- DSB-AM:
  - We have seen that (assume $\phi_p = 0$ for simplicity)

$$\hat{M}(t) = Re\left\{\frac{Y_z(t)}{A}\right\}$$

$$= Re\left\{\frac{x_z(t)}{A} + \frac{W_z(t)}{A}\right\}$$

$$x_z(t) = Am(t)$$

$$= m(t) + \frac{W_I(t)}{A}$$

Fidelity in Analog Communication Systems

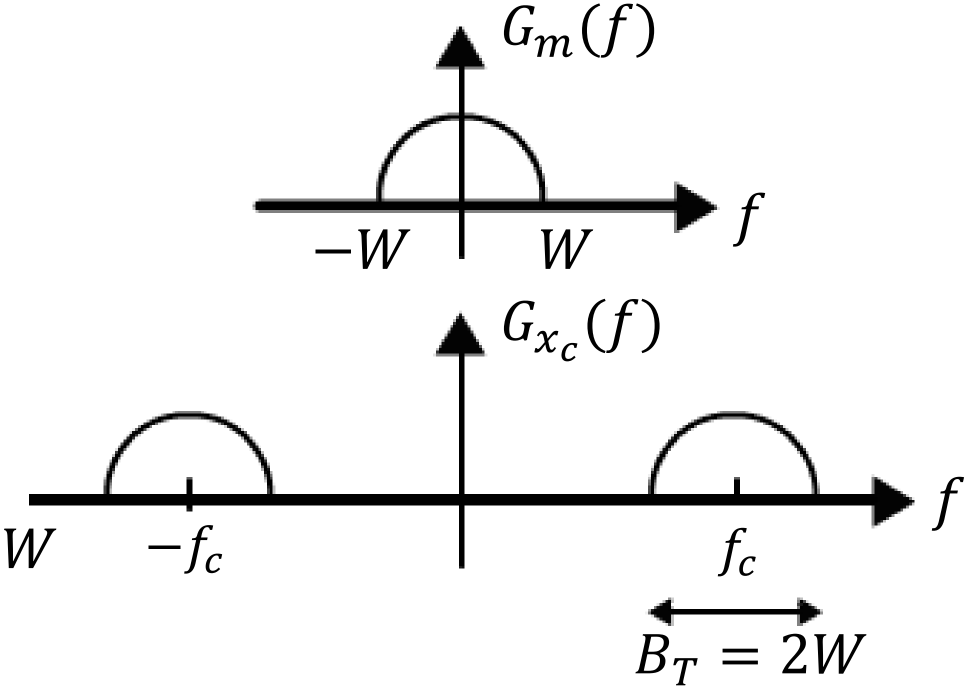
- It follows that the SNR_m is

$$SNR_{m,\,\mathrm{DSB-AM}} = \frac{\text{power message}}{\text{power noise after demodulation}}$$

$$= \frac{P_m}{E\left[\frac{W_I(t)^2}{A^2}\right]} = \frac{A^2 P_m}{\frac{N_0}{2} B_R}$$

$$= \frac{A^2 P_m}{N_0 W}$$

setting
$B_R = B_T = 2W$



Fidelity in Analog Communication Systems

- Transmission SNR:

$$SNR_t = \frac{\text{power received signal}}{\text{power received noise}}$$

$$= \frac{A^2 P_m}{N_0 2W} = \frac{SNR_{m,\,\mathrm{DSB-AM}}}{2}$$

Fidelity in Analog Communication Systems

- Angle modulation:
  - Analysis more complex (see pages 11.12 – 11.19)
  - It can be proved that for FM

$$SNR_{FM} \cong \frac{3K_f^2 P_m}{4\pi^2 W^2} \cdot SNR_{\mathrm{DSB-AM}}$$

hence, $\uparrow K_f \Rightarrow \uparrow SNR_{FM}$

Fidelity in Analog Communication Systems

- Since $B_T \cong 2W\left(1+\frac{K_f|\max m(t)|}{W}\right)$, we have:

$$\uparrow B_T \Rightarrow \uparrow SNR_{FM}$$

… with FM, we can trade bandwidth for fidelity.

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