NJIT

New Jersey's Science & Technology University

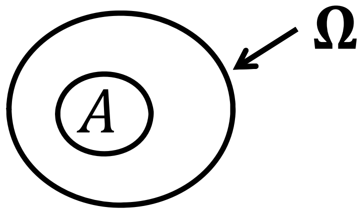
THE EDGE IN KNOWLEDGE

Review of Probability

(Chapter 3)

Concepts to review (see Chapter 3 for details if needed):

- Sample space $\mathbf{\Omega}$
- Event $A \in \mathbf{\Omega}$
- Probability $\Pr(A)$



- Conditional probability $\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

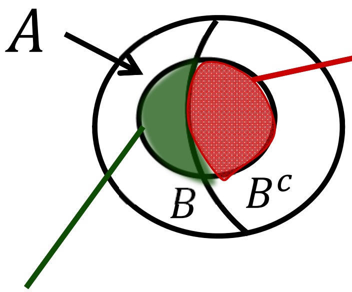
Ex.: $\mathbf{\Omega} = \{1,2,3,4,5,6\}, \Pr(\{i\}) = {}^{1}/_{6}$ for $i = 1,2,\ldots,6$

$$\Pr(\{6\}|\{even\ number\}) = \frac{{}^{1}/_{6}}{{}^{1}/_{2}} = {}^{1}/_{3}$$

Review of Probability

- Law of total probability

$$\Pr(A) = \Pr(B)\Pr(A|B) + \Pr(B^c)\Pr(A|B^c)$$



$\Pr(A \cap B^c) = \Pr(B^c)\Pr(A|B^c)$

$\Pr(A \cap B) = \Pr(B)\Pr(A|B)$

Remark: Can be extended to any partition $B_1, B_2, \ldots, B_m$ with $B_i \cap B_j = \emptyset$ and $B_1 \cup B_2 \cup \cdots \cup B_m = \boldsymbol{\Omega}$

Review of Probability

- Bayes Rule

$$\Pr(B|A) = \frac{\Pr(A|B)\Pr(B)}{\Pr(A)}$$

- Random variable $X = \text{function of elements of } \boldsymbol{\Omega}$

Uppercase letter (lowercase letters are used for the values that a variable can take)

discrete:

- X takes values in a discrete set $\mathcal{X}$
- Described by a probability function $\Pr(X = x)$ for all $x \in \mathcal{X}$

Review of Probability

Ex.: $\mathcal{X} = \{0,1\}, \Pr(X = 1) = p$

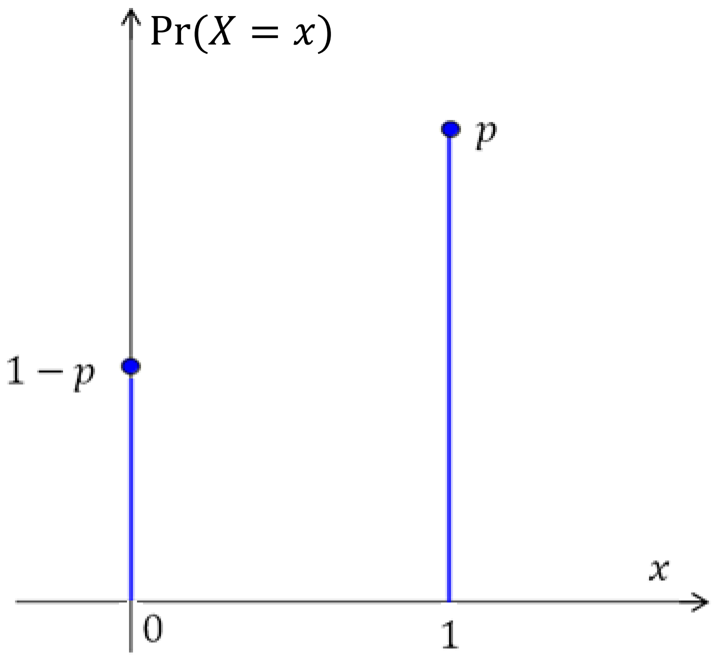
$\Pr(X = 0) = 1 - p$

for some $0 \leq p \leq 1$

$X \sim Ber(p) \longleftarrow p = \Pr(X = 1)$

distributed as

Bernoulli



Review of Probability

- Random variable $X = \text{function of elements of } \boldsymbol{\Omega}$

continuous:

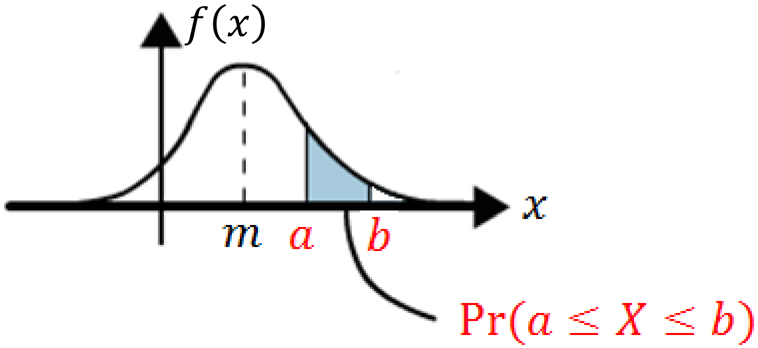
  - X takes values in a continuous set (e.g., $\mathbb{R}$ or $\mathbb{C}$)
  - Described by a probability <u>density</u> function $f(x)$
  - Probabilities are obtained by integrating $f(x)$:

$$\Pr(a \leq X \leq b) = \int_a^b f(x)dx$$

Review of Probability

Ex.: $X \in \mathbb{R}$ (real number)

and $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}}$



Gaussian distribution

$X \sim N(m, \sigma^2)$

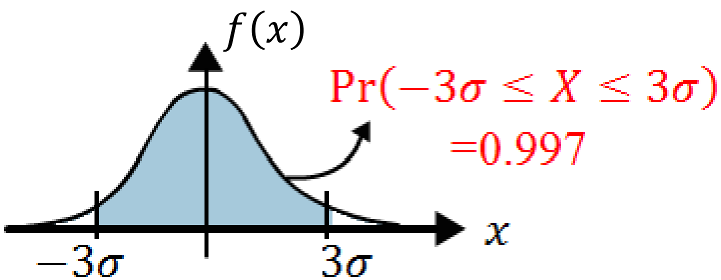
variance

mean

distributed as

Gaussian (Normal)

The spread of the distribution depends on the standard deviation σ:



Review of Probability

- Expectation (or mean)

$$E[X] = \begin{cases} \sum_{x \in X} x \Pr(X = x) & \text{discrete random variables} \\ \int xf(x)dx & \text{continuous random variables} \end{cases}$$

- Variance

$$\mathrm{var}(X) = E[(X - E[X])^2] = \mathrm{E}[X^2] - (E[X])^2$$

Review of Probability

Ex.:

- $Ber(p)$: $E[X] = 0 \cdot (1-p) + 1 \cdot p = p$

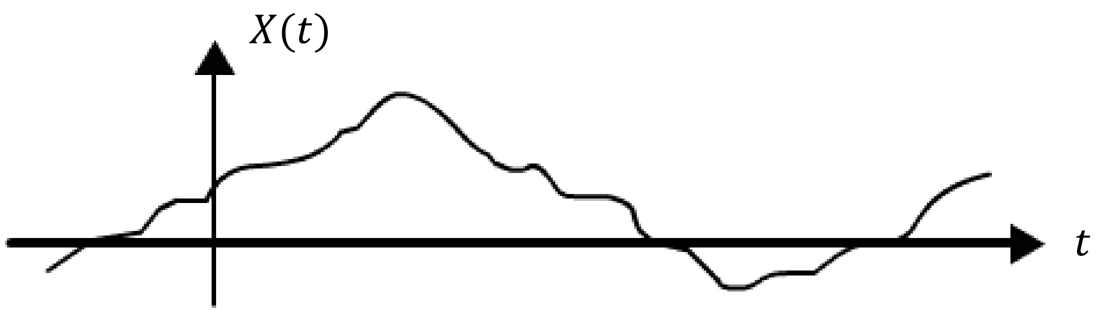
$$\begin{aligned} var(X) &= E[X^2] - p^2 \\ &= 0^2 \cdot (1-p) + 1^2 \cdot p - p^2 \\ &= p(1-p) \end{aligned}$$

- $N(m, \sigma^2)$: $E[X] = m$
  $var(x) = \sigma^2$

Review of Random Processes

(Chapter 9)

- A random process $X(t)$ is a collection of random variables indexed by time t.
- We will be interested in stationary random processes: the probabilistic description of the process does not change with time.



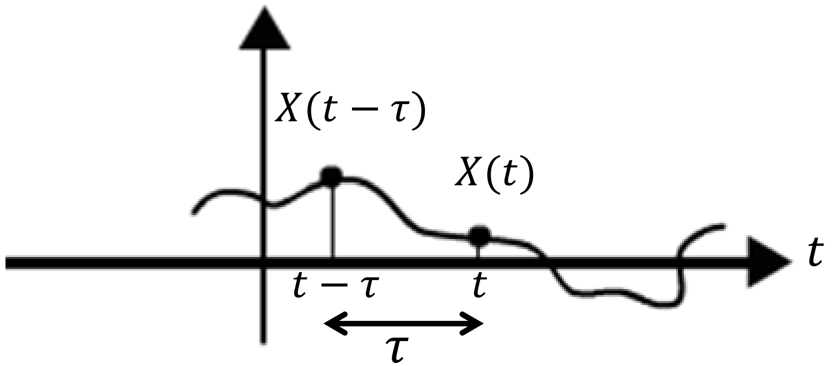
a realization of a random process

- We will be focusing here on real processes.

Review of Random Processes

- Two key quantities that characterize a stationary random process $X(t)$ are:

1) Correlation Function
   - $R_x(\tau) = E[X(t)X(t-\tau)]$

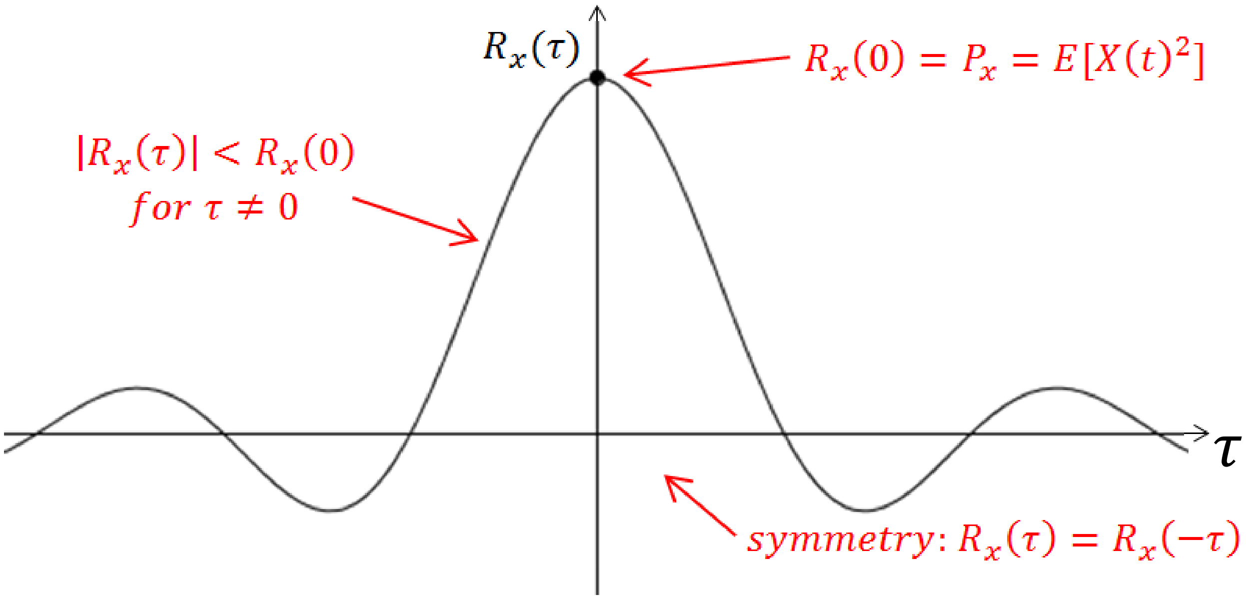


Review of Random Processes

- $R_x(\tau)$ tells us how predictable $X(t)$ is based on $X(t-\tau)$:

  $$\uparrow |R_x(\tau)| \rightarrow \downarrow \text{ "randomness"}$$

- Properties:



Review of Random Processes

2) Power spectral density

- $S_x(f) = \mathcal{F}\{R_x(\tau)\}$
- Real and positive
- Symmetric $(S_x(f) = S_x(-f))$

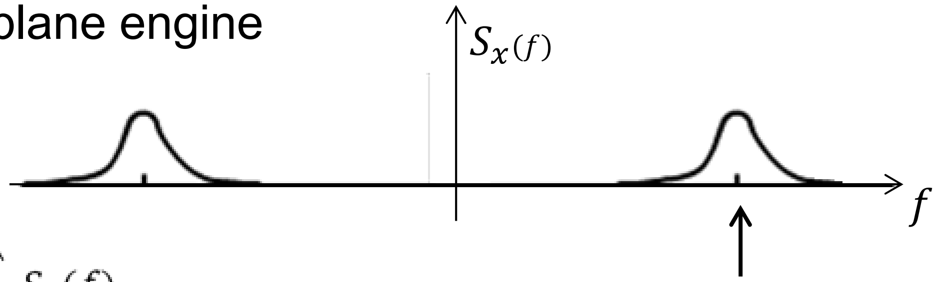
continued on the next slide…

Review of Random Processes

- $S_x(f)$ tells us how much power $X(t)$ has at frequency f:
  $\uparrow$ "spread" of $S_x(f) \rightarrow \uparrow$ rate of change

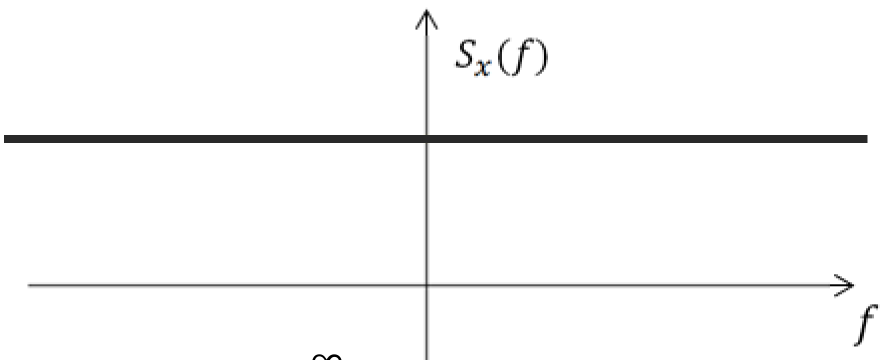
  Ex.:

  a) noise from an airplane engine



frequency of the engine

  b) white noise



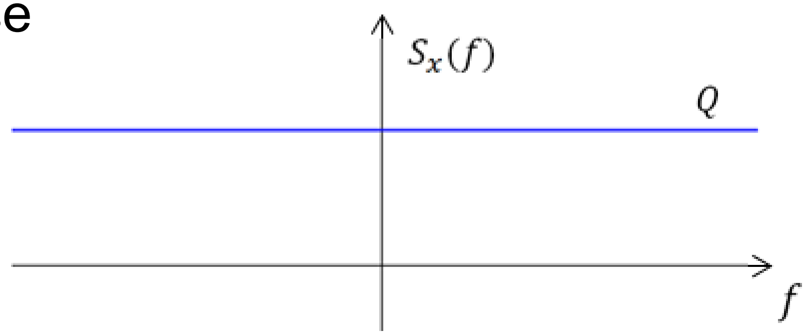
- $P_x = E[X(t)^2] = \int_{-\infty}^{\infty} S_x(f)df$

Review of Random Processes

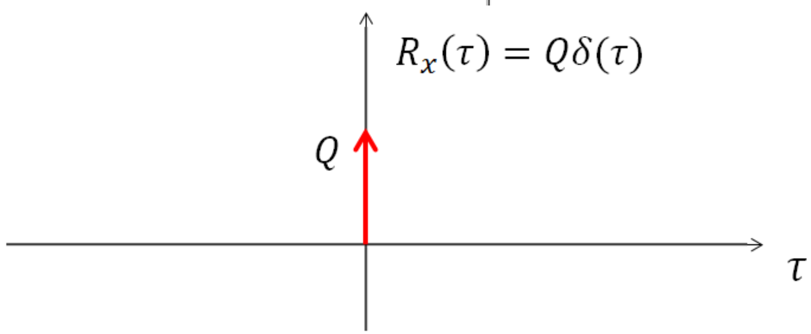
- In communication systems, <u>noise</u> is often modeled as white

$\longrightarrow$ WN: white Noise

$\longrightarrow S_x(f)$



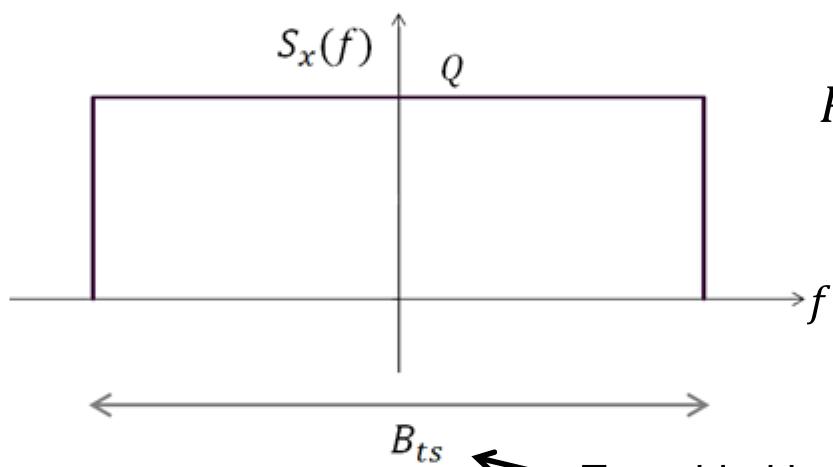
$\longrightarrow R_x(\tau)$



Review of Random Processes

- $P_x = \infty$! WN is a mathematical abstraction

  $\longrightarrow$ we will consider the noise to be white only within a certain bandwidth



$$P_x = B_{ts} \cdot Q$$

Two-sided bandwidth
(includes also negative frequencies)

Review of Random Processes

- More specifically, a noise process $W(t)$ is often modeled as White Gaussian Noise (WGN)

WN with power spectral density $S_w(f) = Q$ within some (two-sided) bandwidth B_{ts}

$+$

Gaussian: $W(t) \sim N(\underset{\substack{\uparrow \\ \text{zero mean}}}{0}, B_{ts}Q)$

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