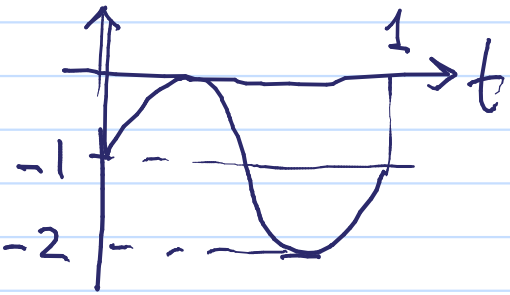
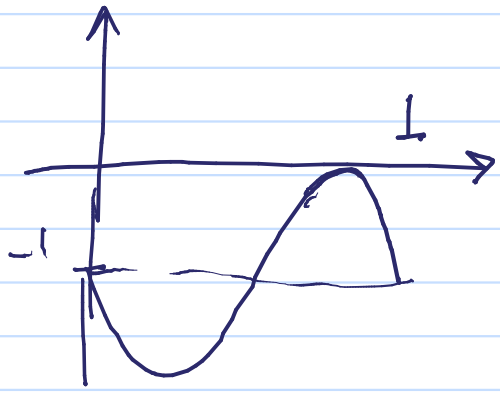
1. a. Effective signal

$$x_e(t) = x_{s,1}(t) - x_{s,0}(t) = \sin(2\pi t) - 1$$



Matched filter

$$h(t) = \sin\left(2\pi(-t+1)\right) - 1$$



b. $\gamma = \frac{E_1 - E_0}{2} = \frac{1/2 - 1}{2} = -\frac{1}{4}$

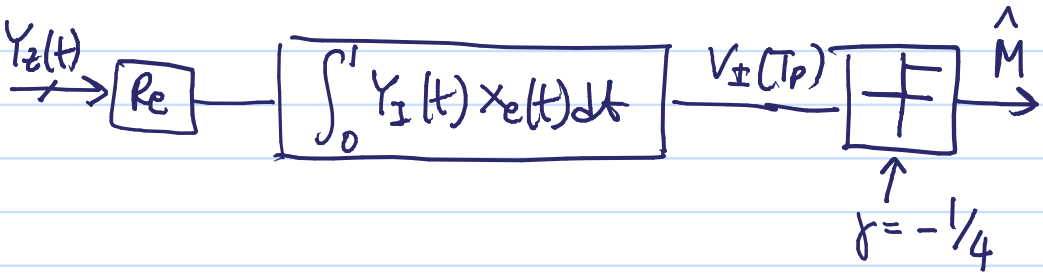
since $E_0 = 1$

$E_1 = 1/2$

c. $P_B(E) = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{\Delta_E(1,0)}{4N_0}}\right)$

where $\Delta_E(1,0) = E_0 + E_1 - 2\int_0^1 \sin(2\pi t)\,dt = \frac{3}{2}$

d.



2. a. $m_0 = -E_0 + \mathrm{Re}(\rho_{10})\sqrt{E_0 E_1} = -2$

$m_1 = +2$

$\sigma^2_{N_I} = \frac{N_0}{2}\Delta_E(1,0) = \frac{N_0}{2}\left(E_0 + E_1 - 2(-1)\right) = 2N_0 = 0.4$

b. $\underbrace{e^{-(0.1-m_0)^2/0.8} \cdot 0.9}_{0.036} \underset{\hat{M}=1}{\overset{\hat{M}=0}{\gtrless}} \underbrace{e^{-(0.1-m_1)^2/0.8} \cdot 0.1}_{0.011}$

$\Rightarrow \hat{M} = 0$

c. $\underbrace{e^{-(0.1-m_0)^2/0.8} \cdot 0.5}_{0.002} \underset{\hat{M}=1}{\overset{\hat{M}=0}{\gtrless}} \underbrace{e^{-(0.1-m_1)^2/0.8} \cdot 0.5}_{0.0055}$

$\Rightarrow \hat{M} = 1$

3. a. For $M=0$,

$$\Delta_E(0,1) = E_0 + E_1 - 2\,\mathrm{Re}(\rho_{01})\sqrt{E_0 E_1}$$

$$= 2 \qquad (\rho_{01} \to 0)$$

$$\Delta_E(0,2) = 4$$

$$\Delta_E(0,3) = 2$$

$$\Rightarrow \{(2,2),(4,1)\}$$

- for $M=1$, $\{(2,3)\}$ since $\int_0^1 \sin(2\pi t)\cos(2\pi t)\,dt = 0$
- for $M=2$, $\{(2,2),(4,1)\}$
- for $M=3$, $\{(2,3)\}$

b. $$P_{WUB}(E) = \frac{1}{4}\left(10 \times \frac{1}{2} \times \mathrm{erfc}\left(\sqrt{\frac{1}{2N_0}}\right) + 2 \times \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{1}{N_0}}\right)\right)$$

c. $X_{2,3}(t) = -X_{2,1}(t)$

In this way, all the conditional distance spectra are

$$\{(2,2),(4,1)\}$$

and the union bound becomes

$$P_{WUB}(E) = \frac{1}{4}\left(8 \times \frac{1}{2} \times \mathrm{erfc}\left(\sqrt{\frac{1}{2N_0}}\right) + 4 \times \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{1}{N_0}}\right)\right)$$

Comparing the union bound approximations, we see that

$$P_{WUB}(E) \simeq \mathrm{erfc}\left(\sqrt{\frac{1}{2N_0}}\right) \quad \text{for the improved system}$$

$$P_{WUB}(E) \simeq \frac{5}{4}\,\mathrm{erfc}\left(\sqrt{\frac{1}{2N_0}}\right) \quad \text{for the original system}$$