1. a. $R_b = \frac{k_b}{T_p} = 1\ Mb/s \Rightarrow T_p = \frac{k_b}{R_b} = 3\times10^{-6}\,s = 3\,\mu s$

Hence, we can choose any 8 waveforms with duration $3\mu s$.

A possible choice is

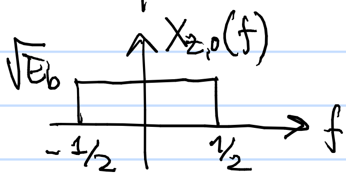
$$x_{z,i}(t) = \begin{cases} e^{j\theta_i} & \text{for} \quad 0 \le t \le T_p \\ 0 & \text{elsewhere} \end{cases}$$

with $\theta_i = \frac{2\pi}{8} i \quad , \quad i = 0, 1, \dots, 7$

b. $\eta_B = \frac{R_b}{B_T} \approx \frac{10^6}{2\times\frac{5}{T_p}} = \frac{\cancel{10^6}}{10} \times 3\times\cancel{10^{-6}} = 0.3$

2. a. $E_0 = E_1 = A^2 \Rightarrow A = \sqrt{E_b}$

b. $m_0 = \mathrm{Re}\left\{ \underbrace{x_{z,0}(t) * h(t)}_{\mathcal{F}^{-1}\{X_{z,0}(f)H(f)\} = \mathcal{F}^{-1}\{X_{z,0}(f)\}} \Big|_{t=T_p} \right\} = \mathrm{Re}\{x_{z,0}(T_p)\} = -\sqrt{E_b}$



$m_1 = \sqrt{E_b}$

$\sigma^2_{N_I} = \frac{N_0}{2} \int |H(f)|^2 df = \frac{N_0}{2} 2 = N_0$

c. $\sigma^2_{N_1} = 0.1$

$m_0 = -1$ and $m_1 = 1$

$f(v_I \mid M=0) = \frac{1}{\sqrt{2\pi 0.1}} e^{-\frac{(v_I+1)^2}{0.2}} \approx 0.003$

$v_I = 0.1$

$f(v_I \mid M=1) = \frac{1}{\sqrt{2\pi 0.1}} e^{-\frac{(v_I-1)^2}{0.2}} \approx 0.022$

$\Rightarrow \frac{f(v_I \mid M=1)}{f(v_I \mid M=0)} = 7.33 > \frac{\pi_0}{\pi_1} = 1 \Rightarrow \hat{M} = 1$

d. We need $\frac{\pi_0}{\pi_1} > 7.33 \Rightarrow \pi_0 > 7.33(1-\pi_0)$

$\Rightarrow \pi_0 (8.33) > 7.33$

$\Rightarrow \pi_0 > \frac{7.33}{8.33} = 0,88$