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THE EDGE IN KNOWLEDGE

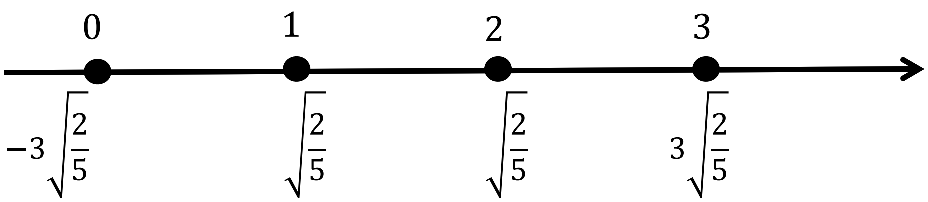
- The union bound $P_{WUB}(E)$ for linear modulations can be easily calculated directly from the constellation.
- The key observation is that the squared Euclidean distance $\Delta_E(i,j)$ can be directly obtained form the constellation, since:

$$\Delta_E(i,j) = \int_{-\infty}^{+\infty} \left| x_{z,i}(t) - x_{z,j}(t) \right|^2 dt$$

$$= E_b \underbrace{\left| d_i - d_j \right|^2}$$

squared Euclidean distance between the points in the constellation

Ex.: a) 4-PAM



$$|d_0 - d_1|^2 = \frac{8}{5}$$

$$|d_0 - d_2|^2 = 4 \cdot \frac{8}{5}$$

$$|d_0 - d_3|^2 = 9 \cdot \frac{8}{5}$$

The distance spectrum is then given as

$$\left[\left\{\frac{8E_b}{5}, 6\right\}, \left\{\frac{32E_b}{5}, 4\right\}, \left\{\frac{72E_b}{5}, 2\right\}\right]$$

and the union bound is

$$P_{WUB}(E) = \frac{1}{4}\left(6\,Q\left(\sqrt{\frac{4E_b}{5N_0}}\right) + 4Q\left(\sqrt{\frac{16E_b}{5N_0}}\right) + 2Q\left(\sqrt{\frac{36E_b}{5N_0}}\right)\right)$$

Union bound approximation → $\simeq \frac{3}{2}\,Q\left(\sqrt{\frac{4E_b}{5N_0}}\right)$

Remark: Comparing with 4-PSK, we see that 4-PSK has a gain of

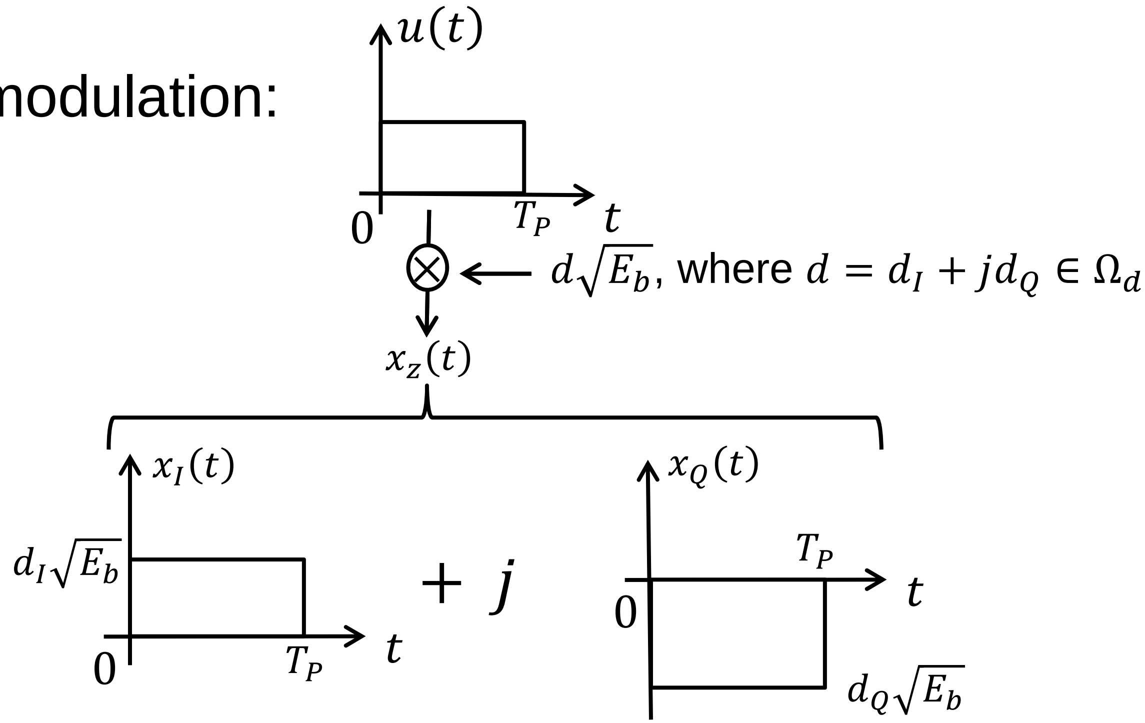
$$\frac{2E_b/N_0}{4E_b/5N_0} = \frac{5}{2} \simeq 3.9\ dB$$

over 4-PAM (why do you think that 4-PAM is not as efficient as 4-PSK?).

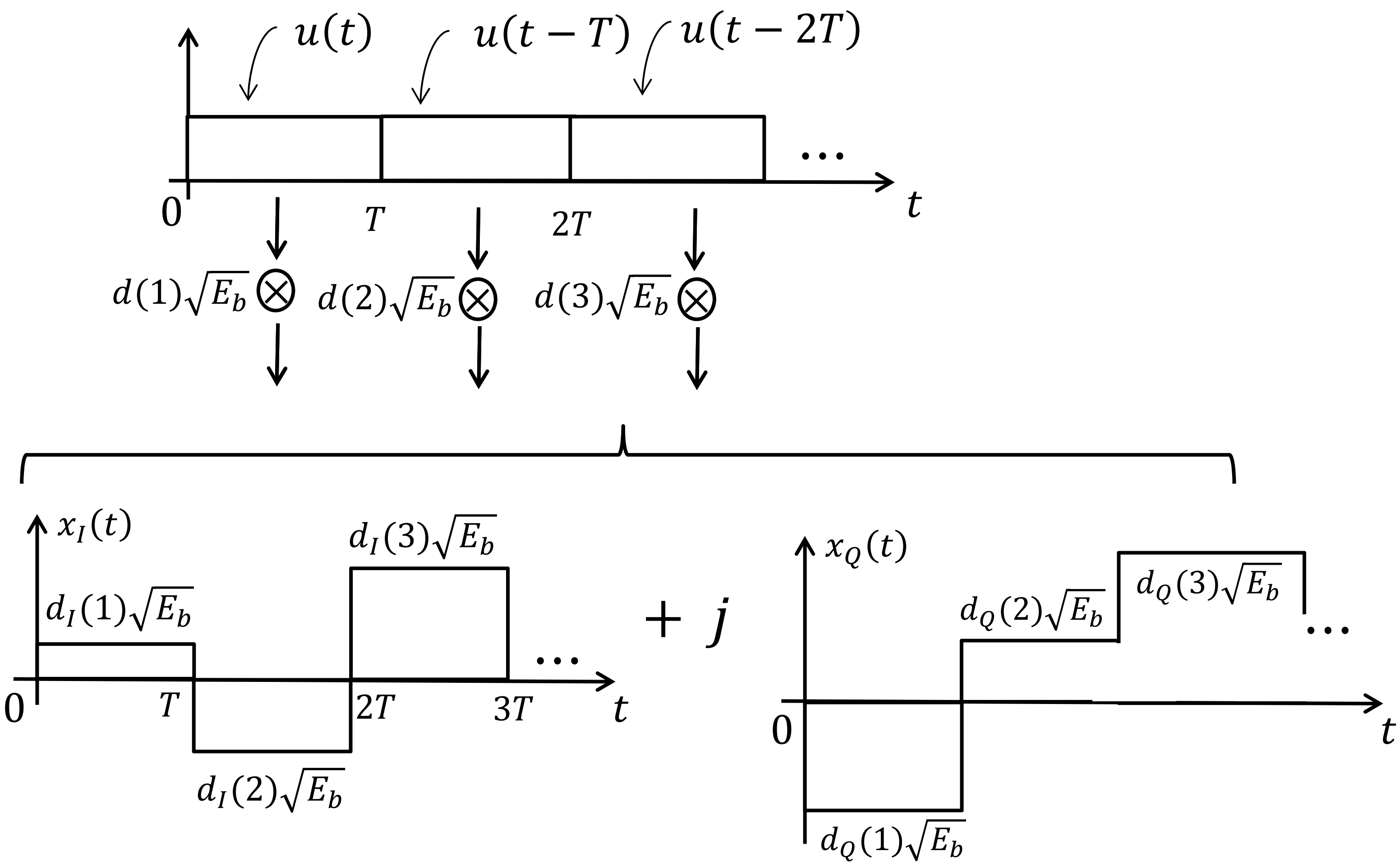
■

Linear stream modulation

- With linear modulation, increasing the number of transmitted bits, namely K_b, requires to use very large constellations (with 2^{K_b} points!)
- This problem is solved by transmitting a stream of linearly modulated symbols.
- Linear modulation:



- Linear stream modulation:



- Each symbol $d(k)$ encodes a different part of the message, say $M(k)$, which carries K_b bits.
- Transmission rate: $R = \frac{K_b}{T_P}$
- Mathematically, a linear stream modulated signal is

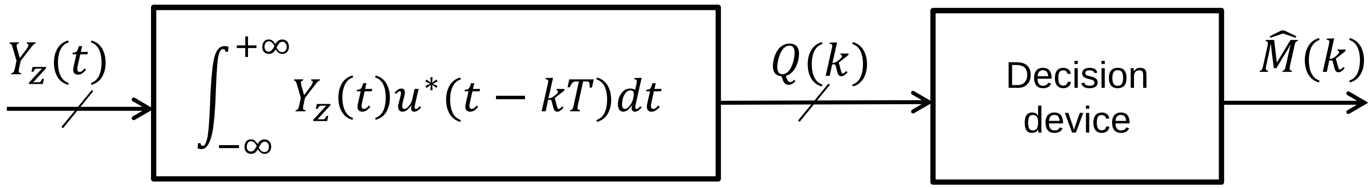
$$x_z(t) = x_I(t) + jx_Q(t)$$

$$= \sum_l d(l)\sqrt{E_b}\,u(t - lT)$$

$$\downarrow$$

$$d(l) = d_I(l) + jd_Q(l)$$

- The MLWD for linear stream modulation requires to perform a correlator for each symbol time (i.e., for each symbol $d(k)$):



- Let us calculate $Q(k)$:

$$Q(k) = \int_{-\infty}^{+\infty} \sum_{l} d(l)\sqrt{E_b}\, u(t-lT)u^*(t-kT)dt$$

$$= d(k)\sqrt{E_b} \underbrace{\int_{-\infty}^{+\infty} |u(t-kT)|^2 dt}_{=1}$$

$$+ \underbrace{\sum_{l \neq k} d(l)\sqrt{E_b} \int_{-\infty}^{+\infty} u(t-lT)u^*(t-kT)dt}_{\text{inter-symbol interference (ISI)}}$$

- In order to have zero ISI, we need to choose $u(t)$ such that

$$\int_{-\infty}^{+\infty} u(t-lT)u^*(t-kT)dt \ = 0 \qquad \text{for all } k \neq l$$

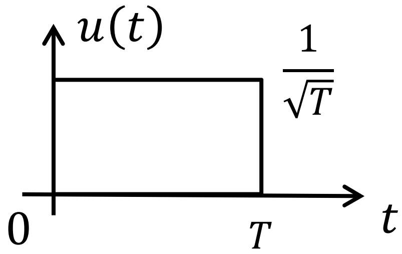
Nyquist criterion for zero ISI

- This condition is equivalent to

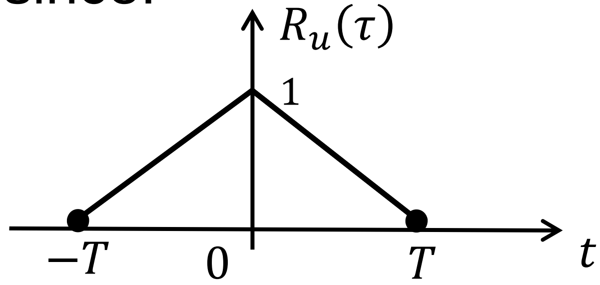
$$R_u(\tau) = 0 \quad \text{for } \tau = kT, k \neq 0$$

correlation function of $u(t)$

Ex.: a)



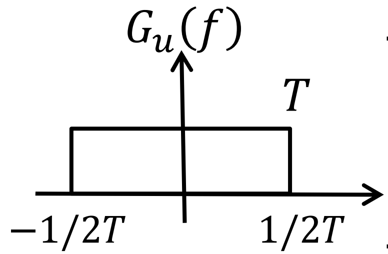
satisfies the Nyquist criterion since:



$$R_u(T) = R_u(-T) = R_u(2T) = \cdots = 0$$

b) $u(t) = \frac{1}{\sqrt{T}} \mathrm{sinc}\left(\frac{t}{T}\right)$ satisfies the Nyquist criterion since

$$R_u(T) = \mathcal{F}^{-1}\left\{ \; \right\} = \mathrm{sinc}\left(\frac{\tau}{T}\right)$$



and $R_u(kT) = 0 \quad \text{for all } k \neq 0$

- See also Fig. 16.5-16.6 and eq. (16.22) in the textbook ■

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