ECE 232 - Circuits and Systems II
Test 1

Please provide clear and complete answers. Don't forget to specify the units of measure!

Part 1

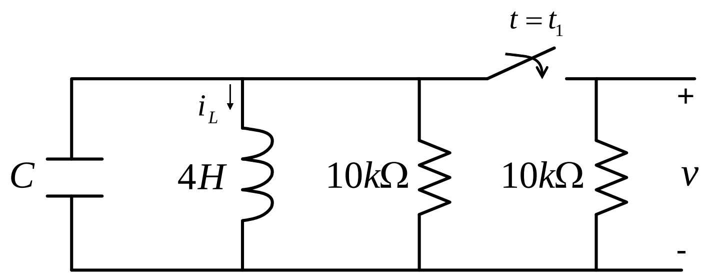

Figure 1:

Consider the circuit in Fig. 1 in which $C = 0$ (this implies that the capacitor behaves as a ...). The switch closes at time $t_1 = 0.4ms$. It is known that $i_L(0^-) = 10mA$.
1.a) Find $i_L(t)$ for $t \geq 0$.
1.b) Plot $i_L(t)$ by providing specific representative values on the time and current axes.

Sol.:
1.a) The circuit is RL with sequential switching. The time constant is

$$\tau_1 = \frac{L}{10k\Omega} = \frac{4}{10^4} = 0.4ms$$

for the time interval $0 \leq t < t_1$ (switch open) and

$$\tau_2 = \frac{L}{10k\Omega||10k\Omega} = \frac{4}{5 \cdot 10^3} = 0.8ms$$

for the time interval $t \geq t_1$ (switch closed). The behavior of the current is described by

$$i_L(t) = \begin{cases} i_L(0)e^{-2500t} & \text{for } 0 \leq t < t_1 = 0.4ms \\ i_L(t_1)e^{-1250(t-t_1)} & \text{for } t \geq t_1 = 0.4ms \end{cases}.$$

We calculate:

$$\begin{aligned} i_L(t_1) &= i_L(0)e^{-2500t_1} \\ &= 10e^{-2500t_1} = 10e^{-1} = 3.67mA, \end{aligned}$$

so that

$$i_L(t) = \begin{cases} 10e^{-2500t} & \text{for } 0 \leq t < t_1 = 0.4ms \\ 3.67e^{-1250(t-4\cdot 10^{-3})} & \text{for } t \geq t_1 = 0.4ms \end{cases} \quad mA,$$

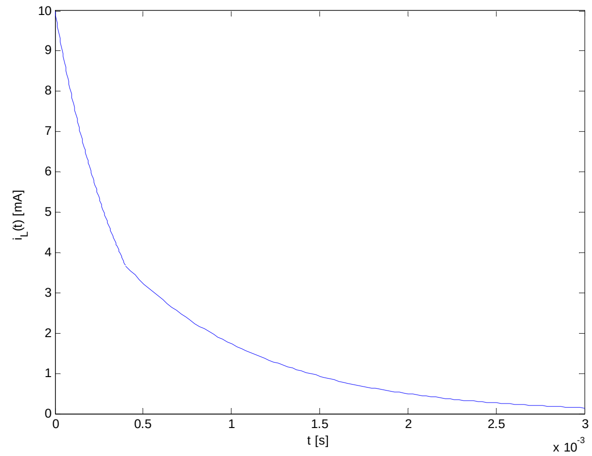

Figure 2:

where t is measured in seconds.
1.b) See figure 2.
Part 2

Consider again the circuit in Fig. 1 but now with $C = 0.1\mu F$. Moreover, the switch is open all the time (i.e., $t_1 = \infty$). It is known that $i_L(0^-) = 10mA$ and $v(0^-) = 0$.
2.a) In what regime is the circuit operating? Find $v(t)$ for $t \geq 0$.
2.b) Plot $v(t)$ by providing specific representative values on the time and voltage axes (if the response is overdamped specify the two time constants, if underdamped specify time constant and damped frequency). Indicate in what invervals the capacitor is charging or discharging.
2.c) (Extra) Find and plot $i_L(t)$ for $t \geq 0$.

Sol.:
2.a) Let us calculate the Neper frequency and resonant frequency

$$\begin{aligned} \alpha &= \frac{1}{2RC} = \frac{1}{2 \cdot 10^4 \cdot 10^{-7}} = 500 \text{ rad/s} \\ \omega_0 &= \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4 \cdot 10^{-7}}} = 1581 \text{ rad/s}. \end{aligned}$$

We have $\omega_0 > \alpha$, so we are in the underdamped regime. The damped frequency is given by

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 1500 \text{ rad/s}.$$

The voltage is then given by

$$v(t) = e^{-500t}\left[B_1 \cos 1500t + B_2 \sin 1500t\right],$$

where B_1 and B_2 are found imposing the initial conditions

$$\begin{aligned} B_1 &= 0 \\ \alpha B_1 - \omega_d B_2 &= -\frac{1}{C}\left[-\frac{v(0)}{R} - i_L(0)\right] \\ &= \frac{i_L(0)}{C} = \frac{10^{-2}}{10^{-7}} = 10^5 \\ &\rightarrow \quad B_2 = -\frac{10^5}{1500} = -\frac{2}{3}10^2 = -66.7. \end{aligned}$$

We finally obtain

$$v(t) = e^{-500t}\left[-66.7 \cdot \sin 1500t\right] \; V.$$

2.b) The period of the oscillation is $4.2ms$, while the time constant of the exponential is $2ms$. See figure 3.

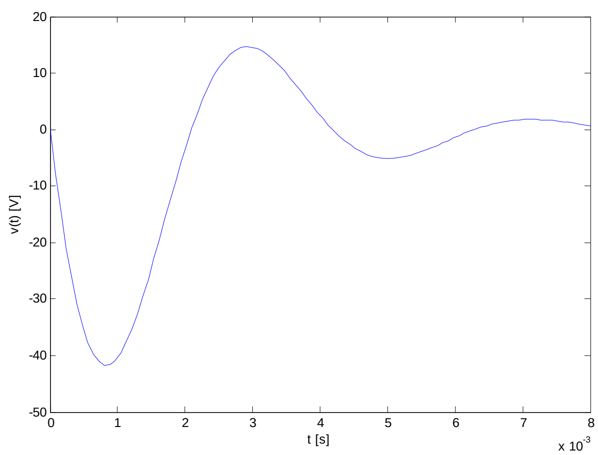

Figure 3:

2.c) There are different ways to answer this question. Perhaps the simplest (no need for integrals or derivatives!) is to recall that the current in the inductor satisfies exactly the same differential equation as the voltage. So, we have

$$i_L(t) = e^{-500t}\left[B_1' \cos 1500t + B_2' \sin 1500t\right]$$

where the initial conditions are given by

$$\begin{aligned} B_1' &= 10^{-2} \\ \alpha B_1 - \omega_d B_2 &= 0 \\ &\rightarrow \quad B_2 = \frac{\alpha B_1}{\omega_d} = \frac{500 \cdot 10^{-2}}{1500} \\ &= \frac{1}{3}10^{-2} = 3.3 \cdot 10^{-3}, \end{aligned}$$

so that

$$i_L(t) = e^{-500t}\left[10 \cos 1500t + 3.3 \sin 1500t\right] \; mA.$$