ECE 232 - Circuits and Systems II
Test 2

Please provide clear and complete answers. Don't forget to specify the units of measure!

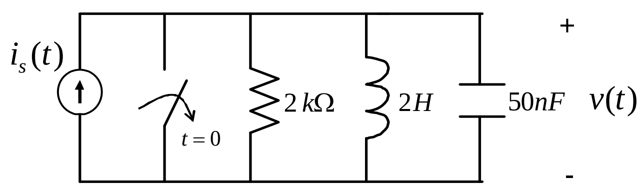

Figure 1:

Consider the circuit in figure 1 and assume that there is no energy stored in the circuit elements at time $t = 0^-$. The switch opens at time $t = 0$. The source provides current $i_s(t) = 10\cos(1000t)\ mA$.

1.a) Find the equivalent circuit in the s-domain and calculate the transfer function between the current input $i_s(t)$ and the voltage $v(t)$.

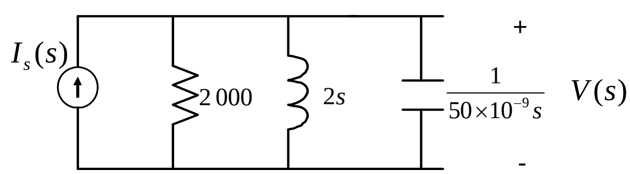

Figure 2:

Sol.: The equivalent circuit is shown in figure 2. The transfer function is given by the impedence of the parallel RLC branches. Therefore, we have

$$\begin{aligned} H(s) &= \frac{V(s)}{I_s(s)} = \left(2\times10^3||2s||\frac{1}{50\times10^{-9}s}\right) \\ &= \frac{1}{0.5\times10^{-3}+\frac{0.5}{s}+50\times10^{-9}s} \\ &= \frac{20\times10^6 s}{s^2+10^4 s+10^7}. \end{aligned}$$

1.b) Find the Laplace transform $V(s)$, calculate and sketch poles and zeros. Which poles correspond to the transient component of the solutions? Which ones to the steady-state?

Sol.: The Laplace transform $V(s)$ is given by

$$\begin{aligned} V(s) &= H(s)I_s(s) \\ &= \frac{20\times 10^6 s}{s^2+10^4 s+10^7}\cdot\frac{10\times 10^{-3}s}{s^2+10^6}, \end{aligned}$$

since $I_s(s)=\frac{10\times 10^{-3}s}{s^2+10^6}$. The poles are given by:

- Source poles: $j10^3$ and $-j10^3$. These contribute to the steady-state component of the solution, which we expect to be a sinusoid with the same frequency (1000 rad/s) of the input;
- Transfer function poles: $-8,873$ and $-1,127$. These contribute to the transient component of the solution, which we expect to be the sum of two decaying exponentials with time constants $1/8,873$ and $1/1,127$, respectively.

1.c) Find $v(t)$ and evaluate the expression for $v(t)$ in steady-state.

Sol.: We use the partial fraction expansion method:

$$V(s)=\frac{K_1}{s+8,873}+\frac{K_2}{s+1,127}+\frac{K_3}{s-j10^3}+\frac{K_3^*}{s+j10^3}.$$

We obtain

$$\begin{aligned} K_1 &= \frac{20\times 10^6 s}{s+1,127}\cdot\frac{10\times 10^{-3}s}{s^2+10^6}|_{s=-8,873} \\ &= -25.5 \\ K_2 &= \frac{20\times 10^6 s}{s+8,873}\cdot\frac{10\times 10^{-3}s}{s^2+10^6}|_{s=-1,127} \\ &= 14.45 \\ K_3 &= \frac{20\times 10^6 s}{s^2+10^4 s+10^7}\cdot\frac{10\times 10^{-3}s}{s+j10^3}|_{s=j10^3} \\ &= 5.52+j4.97 = 7.43e^{j41.98^o}. \end{aligned}$$

It follows that

$$v(t)=(-25.5e^{-8,873t}+14.45e^{-1,127t}+14.86\cos(1000t+41.98^o))u(t),$$

In steady-state, we have

$$v(t)=14.86\cos(1000t+41.98^o)u(t)$$

1.d) Assume now that $v(0^-)=10V$ and repeat 1.b.

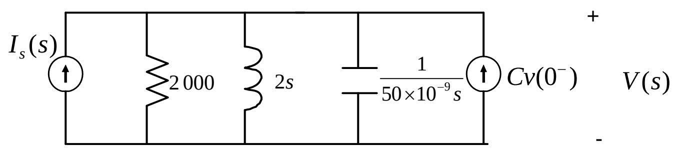

Figure 3:

Sol.: The only difference is that now we have an extra current source in parallel to the capacitor (see figure 3). This source is given as in the figure and equals $I_s'(s) = Cv(0^-) = 50 \times 10^{-9} \times 10 = 50 \times 10^{-8}$. Using the superposition principle, we immediately have

$$\begin{aligned} V(s) &= H(s)(I_s(s) + I_s'(s)) \\ &= \frac{20 \times 10^6 s}{s^2 + 10^4 s + 10^7} \cdot \left(\frac{10 \times 10^{-3} s}{s^2 + 10^6} + 50 \times 10^{-8} \right). \end{aligned}$$

The poles are given as at point 1.b.

1.e) (Extra) For $v(0^-) = 10V$ repeat also 1.c).

Sol.: By the previous point, we have that $v(t)$ is equal to what we calculated at point 1.c) plus the inverse Laplace transform of

$$\begin{aligned} H(s)I_s'(s) &= \frac{20 \times 10^6 s}{s^2 + 10^4 s + 10^7} \cdot 50 \times 10^{-8} \\ &= \frac{K_1}{s + 8,873} + \frac{K_2}{s + 1,127} \end{aligned}$$

Using partial fraction expansion, we calculate

$$\begin{aligned} K_1 &= \frac{10s}{s + 1,127}|_{s=-8,873} = 11.45 \\ K_2 &= \frac{10s}{s + 8,873}|_{s=-1,127} = -1.45, \end{aligned}$$

so that

$$\begin{aligned} v(t) &= (-25.5e^{-8,873t} + 14.45e^{-1,127t} + 14.86\cos(1000t + 41.98^o) \\ &\quad +11.45e^{-8,873t} - 1.45e^{-1,127t})u(t) \\ &= (-14.05e^{-1,127t} + 13e^{-8,873t} + 14.86\cos(1000t + 41.98^o))u(t). \end{aligned}$$