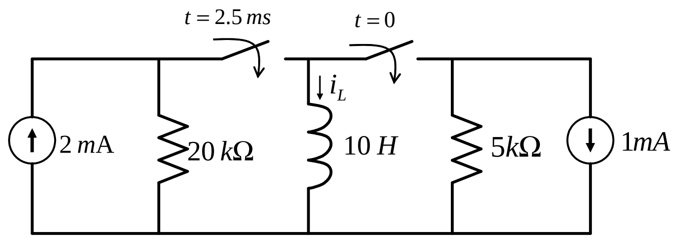
ECE 232 - Circuits and Systems II
Test 1

Please provide clear and complete answers. Don't forget to specify the units of measure!



Consider the circuit in the figure in which the initial energy stored in the inductor is zero.
1.a) Find $i_L(t)$ for $0 \leq t \leq 2.5\ ms$.
1.b) Find $i_L(t)$ for $t \geq 2.5\ ms$.
1.c) Plot $i_L(t)$ for $t \geq 0$.
1.d) Identify the intervals of time in which the inductor is charging or discharging.
1.e) Calculate the overall energy dissipated by the 20 $k\Omega$ resistor.

(Extra: 1.f) Assume that the inductor in the figure is actually the parallel of two inductors both with inductance 20 H. Calculate the current in the two inductors for the interval $0 \leq t \leq 2.5\ ms$)

Sol.:
1.a) For $0 \leq t \leq 2.5\ ms$, the circuit has a time constant of

$$\tau = \frac{L}{R} = \frac{10}{5 \cdot 10^3} = 2\ ms$$

and the final value of the current in the inductor is

$$i_L(\infty) = -1\ mA.$$

Therefore, since the initial value is $i_L(0) = 0$ (no energy is stored initially), we have

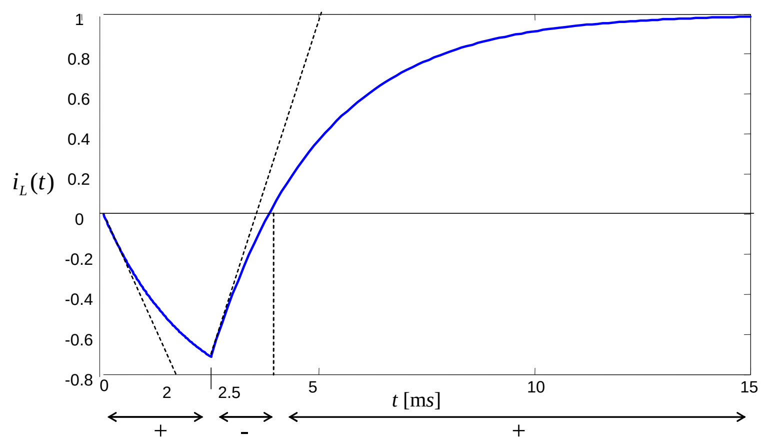
$$i_L(t) = -1 + (0 - (-1))e^{-t/\tau} = -1 + e^{-500t}\ mA.$$

1.b) For $t \geq 2.5\ ms$, the circuit has a time constant of

$$\tau = \frac{L}{R} = \frac{10}{(20k||5k)} = \frac{10}{4 \cdot 10^3} = 2.5\ ms,$$

and the final value of the current in the inductor is

$$i_L(\infty) = 1\ mA.$$



Therefore, since the initial value is

$$i_L(2.5\ ms) = -1 + e^{-500 \cdot 2.5 \cdot 10^{-3}} = -0.71\ mA,$$

we have

$$\begin{aligned} i_L(t) &= 1 + (-0.71 - 1)e^{-(t-2.5\cdot 10^{-3})/\tau} \\ &= 1 - 1.71e^{-400(t-2.5\cdot 10^{-3})}\ mA. \end{aligned}$$

1.c) A plot is shown below.

1.d) Please see figure where "+" stands for charging and "-" for discharging.

1.e) The resistor is only active for $t \geq 2.5\ ms$. The current (from up to down) is

$$i_R(t) = 1 - i_L(t) = 1.71e^{-400(t-2.5\cdot 10^{-3})}\ mA$$

in that interval. The energy dissipated by the resistor is

$$\begin{aligned} w_R &= 20 \cdot 10^3 \int_{2.5\cdot 10^{-3}}^{\infty} (1.71e^{-400(t-2.5\cdot 10^{-3})} \cdot 10^{-3})^2 dt \\ &= 58.48 \cdot 10^{-3} \int_{2.5\cdot 10^{-3}}^{\infty} e^{-800(t-2.5\cdot 10^{-3})} dt \\ &= 58.48 \cdot 10^{-3} \int_0^{\infty} e^{-800t} dt \\ &= \frac{58.48 \cdot 10^{-3}}{(-800)} \left[e^{-800t}\right]_0^{\infty} \\ &= 7.31 \cdot 10^{-5}\ J. \end{aligned}$$

1.f) Since the two parallel inductors have equivalent inductance 10 H, the current flowing in the parallel is $i_L(t) = -1 + e^{-500t}\ mA$ in the given interval. Now, we can find the voltage across the inductor as

$$\begin{aligned} v(t) &= -5000 \cdot (i_L(t) + 10^{-3}) \\ &= -5 \cdot (-1 + e^{-500t} + 1) \\ &= -5e^{-500t}\ V. \end{aligned}$$

The current over the each inductor is thus

$$\begin{aligned} i_1(t) &= i_2(t) = \frac{1}{20}\int_0^t -5e^{-500x}dx \\ &= -0.5 + 0.5e^{-500t} \ mA \\ &= \frac{i_L(t)}{2}, \end{aligned}$$

which is simply half of the overall current flowing over the parallel of the two inductors.