ECE 642 - Final Fall 2016

Please justify all your responses (responses without justifications will not be considered). Please label your axes and plot with car

1. (3 points) We need to design a baseband demodulator for an unusual binary communication system in which the sufficient statistic V_I has the following conditional distributions

$$f(v_I|M=0) = \frac{1}{2}(u(v_I+2) - u(v_I))$$
$$f(v_I|M=1) = \begin{cases} 1 - |v_I| & \text{for } -1 \le v_I \le 1 \\ 0 & \text{otherwise} \end{cases}$$

where $u(v) = 1$ if $v \ge 0$ and $u(v) = 0$ otherwise.

a. Assuming that the probability of both bits is the same, find the threshold of the optimal test.

b. Compute the probability of error for the optimal demodulator described at the previous point.

c. Assume now that the bit probabilities are given as $\pi_0 = 2/3$ and $\pi_1 = 1/3$. Compute the optimal threshold.

2. (5 points) Consider the two waveforms

$$x_{z,0}(t) = A\text{sinc}(2t)$$
$$x_{z,1}(t) = \frac{A}{\sqrt{2}}(1+j)\text{sinc}(2t)$$

transmitted with the same probability $\pi_0 = \pi_1 = 1/2$.

a. Compute A as a function of E_b.

b. The baseband demodulator uses a filter $H(f) = j$ for $-1/2 \le f \le 1/2$ and $H(f) = 0$ along with the optimal threshold γ. Compute m_0, m_1 and $\sigma^2_{N_I}$, that is, the mean values and the variance of the sufficient statistics. To do this, assume a sampling time $T_p = 0$.

c. Compute the probability of error for the baseband demodulator of the previous point as a function of E_b/N_0.

d. Evaluate the impuse response of the matched filter (you can set $T_p = 0$ as the sampling time).

e. Compare the probability of error obtained at point c with that obtained with the matched filter. How many dBs are gained with the matched filter.

3. (2 points) Consider a linear modulation scheme with $K_b = 3$ and constellation $\Omega_d = \{3A \pm jA, -3A \pm jA,\ A \pm j3A, -A \pm j3A\}$.

a. Compute A.

b. Compute the union bound approximation as a function of E_b.