ECE 642 - Final Spring 2016

Please justify all your responses (responses without justifications will not be considered). Please label your axes and plot with care.

1. (3 points) We are given a binary communication system in which the sufficient statistic is given as

$$V = m_i + W,$$

where $m_0 = 0$ is the signal received when the message is $M = 0$ and $m_1 = 1$ when $M = 1$; and the noise W has the following pdf (it is not Gaussian!):

$$p(w) = \begin{cases} 1 - |w| & \text{for } -1 \leq w \leq 1 \\ 0 & \text{otherwise} \end{cases}.$$

a. Assuming that the probability of both bits is the same, find the threshold of the optimal test that has V as its input.

b. Compute the probability of error for the optimal demodulator described at the previous point.

c. Assume now that the bit probabilities are given as $\pi_0 = 3/4$ and $\pi_1 = 1/4$. Compute the optimal threshold.

2. (4 points) Consider the two waveforms

$$\begin{aligned} x_{z,0}(t) &= A\text{rect}(t) \\ x_{z,1}(t) &= A(1+j)\text{rect}(t) \end{aligned}$$

where $rect(t) = u(t) - u(t-1)$, with $u(t)$ being the unit step function ($u(t) = 1$ for $t \geq 0$ and $u(t) = 0$ for $t < 0$). Assume that $\pi_0 = \pi_1 = 1/2$.

a. Compute A as a function of E_b.

b. Detail the single-correlator optimal demodulator by specifying all the operations and simplifying the diagram as much as possible.

c. Compute the probability of error for the optimal demodulator.

d. What is the loss of the given modulation scheme as compared to BPSK?

3. (2 points) We are given a system that transmits $K_b = 3$ bits with the following waveforms

$$x_{z,i}(t) = \begin{cases} Ae^{j2\pi it} & \text{for } 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

for $i = 0, 1, ..., 7$. Assume that the probability of all waveforms is the same.

a. Compute A as a function of E_b.

b. Compute the conditional distance spectrum for all messages.

4. (1 point) For a linear stream modulation system with symbol period T does the waveform $u(t) = \frac{1}{\sqrt{T/2}}\text{sinc}(\frac{2t}{T})$ satisfy the Nyquist criterion for zero inter-symbol interference?