**ECE 642 - Final Spring 2013**

Please justify all your responses (responses without justifications will not be considered). Please label your axes and plot with care.

**1. (3 points)** Consider MSK waveforms (i.e., BFSK with frequency deviation $f_d = 1/(4T_p)$) with $E_b = 1$ and $T_p = 1$. The probability of bit 0 is $\pi_0 = 1/2$.

a. Detail an optimal demodulator (the simpler the better!).

b. If $N_0 = 0.1$, what is the numerical value of the probability of bit error (you can use an approximation)?

c. Calculate the plot the energy spectral densities $G_{x_{z.0}}(f)$ and $G_{x_{z,1}}(f)$.

**2. (4 points)** Consider a pulse position modulation (PPM) system used for the transmission of $K_b = 2$ bits. Recall that a PPM system is such that the waveforms $x_{z,i}(t)$ for $i = 0, 1, 2, 3$ occupy equal and disjoint portions of the interval $[0, T_p]$.

a. Write the equations of the waveforms $x_{z,i}(t)$ as a function of $E_b$ and $T_p$.

b. Detail an optimal demodulator (the simpler the better!).

c. Evaluate the conditional distance spectrum for each one of the four messages.

d. Calculate the union bound on the probability of word error.

**3. (4 points)** We are given an optimal demodulator for BPSK (consider the single correlator structure for simplicity). Recall that the BPSK waveforms can be written as $x_{z,i}(t) = d_i\sqrt{E_b}u(t)$, $i = 0, 1$, where $u(t)$ is a unit energy rectangle of duration $T_p$, $d_0 = -1$ and $d_1 = 1$.

Unfortunately, due to hardware problems, the transmitted waveforms are such that $d_0 = e^{j\frac{7\pi}{8}}$ instead of $d_0 = -1$ (we instead correctly have $d_1 = 1$). As always, define as $V_I(T_P) = m_{0/1} + N_I$ as the sufficient statistics for this pair of waveforms using the optimal demodulator for BPSK.

a. Calculate $m_0$ and $m_1$.

b. Calculate $\sigma^2_{N_I}$.

c. Evaluate the probability of error as a function of $E_b$ and $N_0$.

d. What is the loss in dB of the given pair of waveforms as compared to BPSK?

**4. (2 points)** Consider the 16 QAM modulation. Recall that 16 QAM is a linear modulation with $d_i = d_{Ii} + jd_{Qi}$ and $d_{Ii}, d_{Qi} \in \{-3A, -A, A, 3A\}$.

a. Find $A$ such that the average energy of the constellation is equal to $4E_b$.

b. Calculate the union bound approximation to the probability of error.