1. MSK · $E_b = 1$, $T_p = 1$

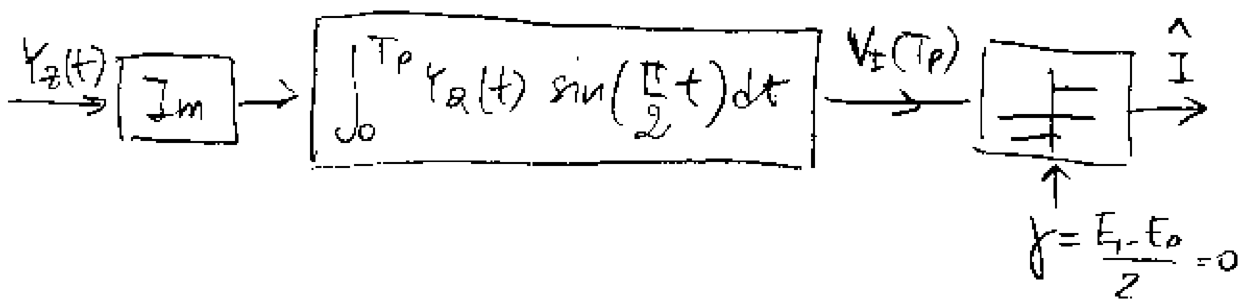
$$X_{z,0}(t) = \begin{cases} e^{j\frac{\pi}{2}t} & \text{for } 0 \le t \le T_p \\ 0 & \text{elsewhere} \end{cases}$$

$$X_{z,1}(t) = \begin{cases} e^{-j\frac{\pi}{2}t} & \text{for } 0 \le t \le T_p \\ 0 & \text{elsewhere} \end{cases}$$

a. Using the correlator structure

$$V_I(T_p) = \mathrm{Re}\left\{ \int_0^{T_p} Y_z(t) \underbrace{\left( e^{j\frac{\pi}{2}t} - e^{-j\frac{\pi}{2}t} \right)}_{2j \sin(\frac{\pi}{2}t)} dt \right\}$$

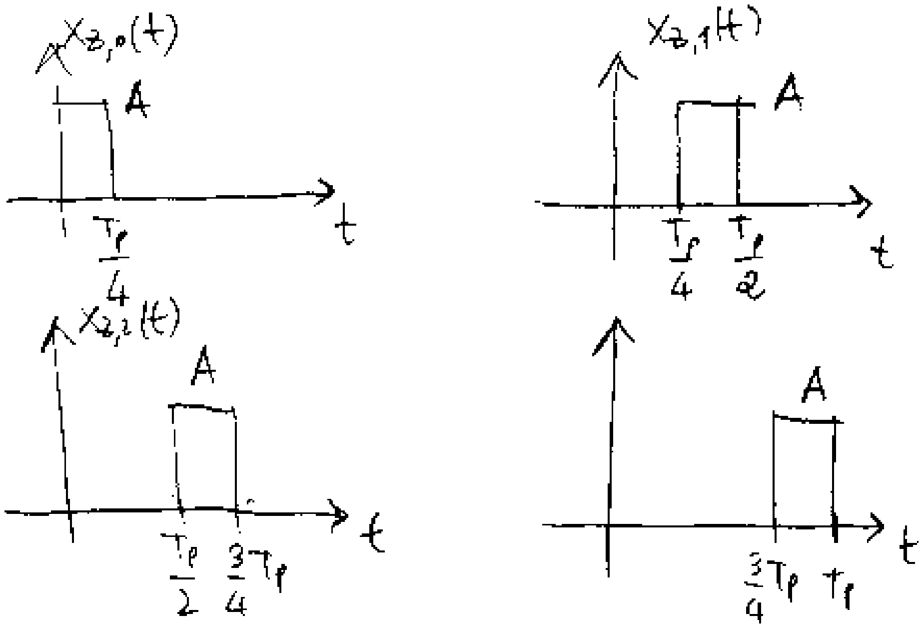
$$= -2 \int_0^{T_p} Y_Q(t) \sin\left(\frac{\pi}{2}t\right) dt$$




b. $$P_B(E) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{5}\right) \simeq \frac{1}{2\sqrt{\pi 5}} e^{-5}$$
$$= 8.5 \times 10^{-4}$$

c. $$G_{X_{z,0}}(f) = E_b T_p \cdot \mathrm{sinc}((f - f_d)T_p) = \mathrm{sinc}\left(f - \frac{1}{4}\right)$$
$$G_{X_{z,1}}(f) = \mathrm{sinc}\left(f + \frac{1}{4}\right)$$
$$D_{X_z}(f) = \frac{1}{2}\mathrm{sinc}\left(f - \frac{1}{4}\right) + \frac{1}{2}\mathrm{sinc}\left(f + \frac{1}{4}\right)$$

2. a.




A is found by imposing that $E_s = 2E_b$:

$$E_s = E_0 = E_1 = E_2 = E_3 = A^2 \frac{T_p}{4} = 2E_b$$

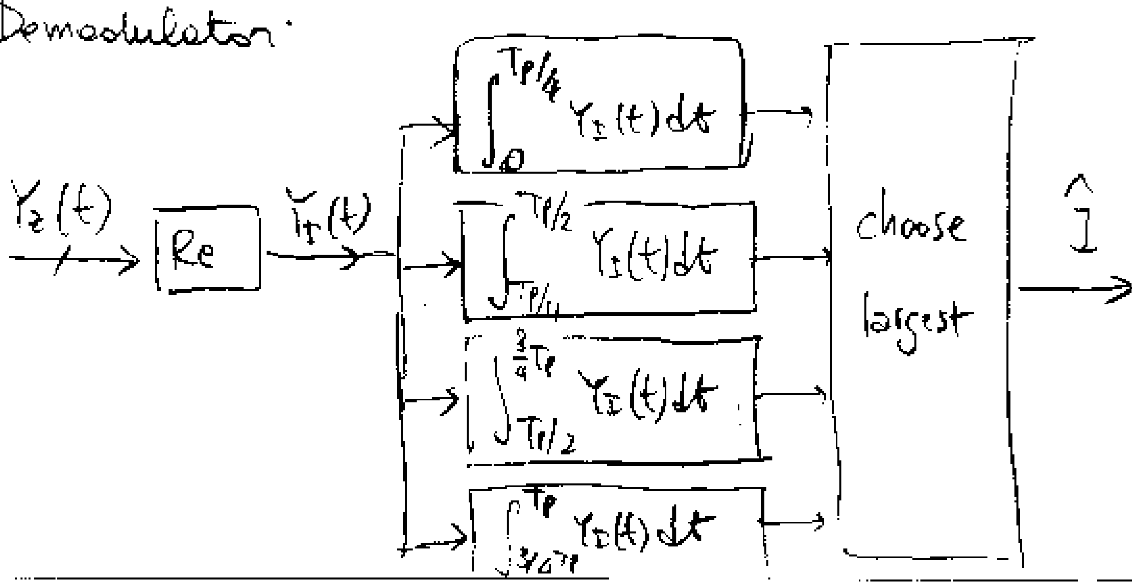
$$\Rightarrow A = \sqrt{\frac{8E_b}{T_p}}$$

b. The opt demodulator is based on the likelihood metrics

$$T_i = A \operatorname{Re}\left\{ \int_{I_i} Y_z(t)\, dt \right\} - \frac{2E_b}{2} \quad \text{where } I_i = \begin{cases} \left[0, \frac{T_p}{4}\right], & i=0 \\ \left[\frac{T_p}{4}, \frac{T_p}{2}\right], & i=1 \\ \left[\frac{T_p}{2}, \frac{3T_p}{4}\right], & i=2 \\ \left[\frac{3}{4}T_p, T_p\right], & i=3 \end{cases}$$

$$= \sqrt{\frac{8E_b}{T_p}} \int_{I_i} Y_I(t)\, dt - E_b$$

$\Rightarrow$ Demodulator:



c. The squared Euclidean distance between any two waveforms is the same and equal

$$\Delta_E(i,j) = 2A^2 \frac{T_p}{4} = A^2 \frac{T_p}{2} = \frac{\overset{4}{8} E_b}{\cancel{T_p}} \frac{\cancel{T_p}}{\cancel{2}}$$

$$= 4E_b$$

$\Rightarrow$ The conditional distance spectrum for all messages is the same and equal to

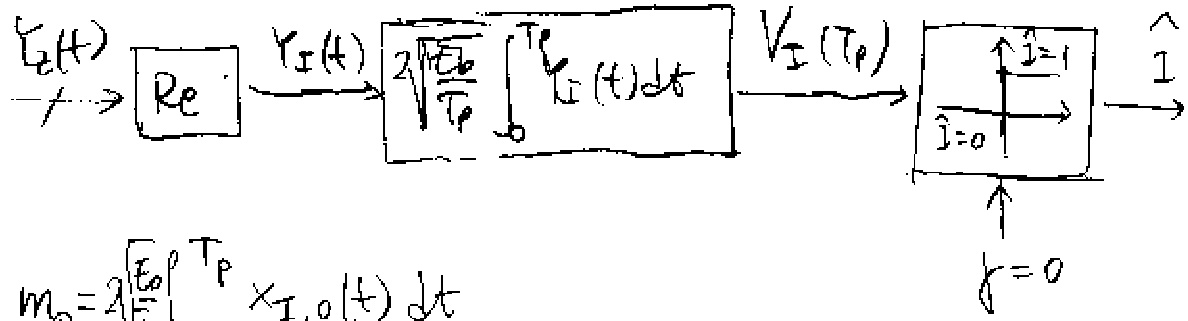
$$A_{d,i} : \{ 4E_b, 3 \}$$

d. Union bound:

$$P_{WUB}(E) = \frac{1}{4} \cdot \underset{\substack{\uparrow \\ \text{\# of pairs} \\ \text{at distance } 4E_b}}{12} \cdot \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_o}}\right)$$

$$= \frac{3}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_o}}\right)$$

3. Optimal demodulator for BPSK:




a. $$m_0 = 2\sqrt{\frac{E_b}{T_p}}\int_0^{T_p} x_{I,0}(t)\,dt$$

$$= 2\sqrt{\frac{E_b}{T_p}}\int_0^{T_p} \cos\left(\frac{7}{8}\pi\right)\sqrt{E_b}\,\frac{1}{\sqrt{T_p}}\,dt$$

$$= 2E_b\cos\left(\frac{7}{8}\pi\right)$$

$$m_1 = 2\sqrt{\frac{E_b}{T_p}}\int_0^{T_p} x_{I,1}(t)\,dt = 2E_b$$

b. $$\sigma^2_{N_I} = \frac{N_0}{2}\int_0^{T_p} |h(t)|^2\,dt = \frac{N_0}{\cancel{2}}\,\overset{2}{\cancel{4}}\,\frac{E_b}{\cancel{T_p}}\,\cancel{T_p} = 2E_bN_0$$

$$h(t) = \begin{cases} 2\sqrt{\frac{E_b}{T_p}} & 0 \le t \le T_p \\ 0 & \text{otherwise} \end{cases}$$

c. $$P_B(E) = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{(m_1 - m_0)^2}{8\sigma^2_{N_I}}}\right)$$

$$= \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{\left(2E_b\left(1-\cos\frac{7}{8}\pi\right)\right)^2}{8\,(2E_bN_0)}}\right)$$

$$= \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b\left(1-\cos\frac{7}{8}\pi\right)^2}{4N_0}}\right)$$

d. With BPSK, we have $P_B(E) = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$

$$\Rightarrow \text{loss} = -10\log_{10}\left(\frac{\left(1-\cos\left(\frac{7}{8}\pi\right)\right)^2}{4}\right) = 0.337\ \text{dB}$$

4. a. We need to impose that

$$\frac{1}{16}\sum_{i=0}^{15}|d_i|^2 = 4$$

$$\Rightarrow \frac{1}{16}\left(4\frac{A^2}{2}+4\frac{9}{2}A^2+8\left(\frac{A^2}{4}+\frac{9}{4}A^2\right)\right)$$

$$=\frac{5}{2}A^2=4 \Rightarrow A=2\sqrt{\frac{2}{5}}$$

b. Minimum distance

$$\Delta_E(\min) = E_b A^2 = \frac{8E_b}{5}$$

$\rightarrow$ Union bound approximation

$$P_W(E) \simeq \frac{1}{16}\left(4.2+8.3+4.4\right)\operatorname{erfc}\left(\sqrt{\frac{8E_b}{20N_0}}\right)$$

$$=\frac{3}{2}\operatorname{erfc}\left(\sqrt{\frac{2E_b}{5N_0}}\right)$$