

ECE 642 - Final Spring 2014

Please justify all your responses (responses without justifications will not be considered). Please label your axes and plot with care.

1. (4 points) Consider the two waveforms defined as $x_{z,0}(t) = 1$ and $x_{z,1}(t) = \sin(2\pi t)$ in the interval $t \in [0, 1]$ and zero elsewhere.

- a. Evaluate and plot the matched filter in the time domain.
- b. Calculate the optimal threshold assuming that the matched filter is used and that $\pi_0 = 1/2$.
- c. Calculate the bit error probability as a function of N_0 .
- d. Draw the simplest decoder structure using a single correlator.

2. (3 points) A digital message M has probability $\pi_1 = 0.1$. Rectangular waveforms with unit energy and opposite signs are used to communicate this message. The matched filter is implemented at the receiver side for decoding. We have $N_0 = 0.2$.

- a. Calculate m_0 , m_1 and $\sigma_{N_I}^2$.
- b. If the sufficient statistics is $V_I(T_P) = 0.1$, what is the optimal (MAPBD) estimate \hat{M} ?
- c. How does the answer change if $\pi_1 = 0.5$?

3. (3 points) You are given the waveforms $x_{z,0}(t) = 1$, $x_{z,1}(t) = \sqrt{2}\sin(2\pi t)$, $x_{z,2}(t) = -1$ and $x_{z,3}(t) = \sqrt{2}\cos(2\pi t)$ in the interval $t \in [0, 1]$ and zero elsewhere.

- a. Evaluate the conditional distance spectrum for all the messages.
- b. Evaluate the union bound.
- c. Suggest an improvement of the set of waveforms by changing only one of the waveforms. Calculate the union bound for this new set and compare with the original set by using the union bound approximation.