ECE 642 - Final Fall 2013

Please justify all your responses (responses without justifications will not be considered). Please label your axes and plot with care.

1. (7 points) Consider a binary communication system with $\pi_0 = \pi_1 = 1/2$, where

$$x_{z,0}(t) = \begin{cases} A & \text{for } 0 \leq t \leq T_p/2 \\ 0 & \text{for } T_p/2 < t \leq T_p \end{cases}$$

and

$$x_{z,1}(t) = \begin{cases} 0 & \text{for } 0 \leq t \leq T_p/2 \\ A & \text{for } T_p/2 < t \leq T_p \end{cases}$$

a. Find A as a function of E_b and T_p.

b. Derive the impulse response of the matched filter.

c. Detail an optimal demodulator based on a single correlator, a "Re" operator and a test threshold in the simplest form you can obtain.

d. Calculate m_0, m_1 and $\sigma^2_{N_I}$ as a function of E_b.

e. Assume that the received baseband signal is $Y_z(t) = X_z(t)e^{j\theta} + W_z(t)$, where θ is a phase and $W_z(t)$ is the usual WGN. Calculate the sufficient statistics $V_I(T_p)$ for the demodulator derived at the previous point when $M = 0$ and when $M = 1$ by detailing the signal and noise components.

f. Based on the result at the previous point, calculate the probability of bit error as a function of E_b/N_0 and θ.

g. What is the loss (in dB) of this scheme as compared to the optimal bit error probability for binary transmission if $\theta = \pi/4$?

2. (4 points) Consider a linear modulation scheme with constellation $\Omega = \{2A, jA, -2A, -jA\}$.

a. Calculate A.

b. Evaluate the conditional distance spectrum for each one of the four messages.

c. Calculate the union bound on the probability of word error.

d. Find the union bound approximation.

3. (1 point) Consider a linear modulation scheme with constellation $\Omega = \{-3Aj, -Aj, Aj, 3Aj\}$.

a. Find the decision regions of the MLWD in the complex plane of the variable Q at the output of the correlator.