ECE 642 - Assignment 8

1. (6 points) Consider the waveforms

$$x_{z,0}(t) = \begin{cases} A & \text{for } 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

and

$$x_{z,1}(t) = \begin{cases} -A & \text{for } 0 \leq t < 1/3 \\ A & \text{for } 1/3 \leq t < 2/3 \\ -A & \text{for } 2/3 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

where $A \geq 0$. Assume that $\pi_0 = 1/2$.

a. Calculate A as a function of E_b.

b. Calculate the correlation between $x_{z,0}(t)$ and $x_{z,1}(t)$ as a function of E_b.

c. Compute the matched filter and the optimal threshold.

d. Detail the single correlator implementation of the optimum baseband demodulator by specifying the operation of each block. Simplify as much as possible.

e. Detail the implementation with two correlators of the optimum baseband demodulator by specifying the operation of each block.

f. Evaluate the probability of error for the optimal baseband demodulator as a function of E_b and N_0.

---

2. (4 points) Consider the binary communication system which uses the waveforms

$$x_{z,0}(t) = \sqrt{E_b/2}\text{sinc}(t) + j\sqrt{E_b/2}\text{sinc}(t)$$

and

$$x_{z,1}(t) = -j\sqrt{E_b}\text{sinc}(t).$$

a. For the filter

$$H(f) = \begin{cases} j & \text{for } -1/2 \le f \le 1/2 \\ 0 & \text{otherwise} \end{cases}$$

compute m_0, m_1 and $\sigma^2_{N_I}$ as a function of E_b and N_0 (as sampling period, choose time zero).

b. Assuming $\pi_0 = 1/2$ and $E_b = 1$, if the sufficient statistic is equal to $V_I = 2.3$, what is the bit value output by the demodulator that uses the filter above and the optimal threshold?

---

3. (**OPTIONAL, NOT GRADED**) In this exercise, we verify the formula we found for the bit error probability, namely $P_b(E) = Q(\frac{m_1 - m_0}{2\sigma_{N_I}})$ for $\pi_0 = \pi_1 = 1/2$, via some numerical simulations. To this end, we are going to generate many random bits with $\pi_0 = \pi_1 = 1/2$ and evaluate the fraction of bits that are incorrectly received. When averaging over a large number of bits, we expect this fraction to be close to $P_b(E)$.

Fix $m_0 = -1$ and $m_1 = 1$ and assume that the noise variance is $\sigma^2_{N_I} = 0.8$.

a. What is the optimal γ?

b. Generate the signal $V_I = m + N_I$ observed after the sampler when m is randomly chosen as m_0 or m_1 with equal probability (i.e., $\pi_0 = \pi_1 = 1/2$). Recall that $N_I \sim \mathcal{N}(0, \sigma^2_{N_I})$. Obtain the optimal bit estimate. Is it correct?

```
sigma2=...;
gamma=...;
m=...; %generate a random bit
VI=...;
estimatedm=...; % this estimate should be -1 if VI is below gamma
and 1 viceversa
error=(m~=estimatedm) % the error is one if the estimate is wrong
and zero otherwise
%What happens if you repeat the code above a few times?
```

c. Now we generate many trasmitted bits and corresponding signals V_I. Calculate the fraction of bits incorrectly decoded. Compare with $P_b(E) = Q(\frac{m_1 - m_0}{2\sigma_{N_I}})$.

ooo

```
N=1000; %number of bits
for i=1:N
m=...;
VI=...;
estimatedm=...; % this estimate should be -1 if VI is below gamma and 1 viceversa
error(i)=(m~=estimatedm); % the error is one if the estimate is wrong and zero otherwise
end
estimatedBEP=sum(error)/N; %this is the fraction of errors (why?)
realBEP=...;
%Compare them!  What happens if we increase N (e.g., try with N=10,000 and then N=100,000)?  Why?
```