1)

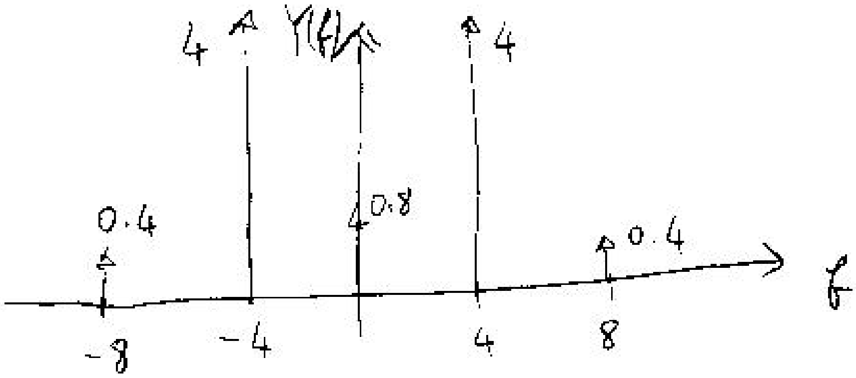
a. $y(t) = 2\left(4\cos(8\pi t)\right) + 0.1\left(4\cos(8\pi t)\right)^2$

$= 8\cos(8\pi t) + 1.6\left(\cos(8\pi t)\right)^2$

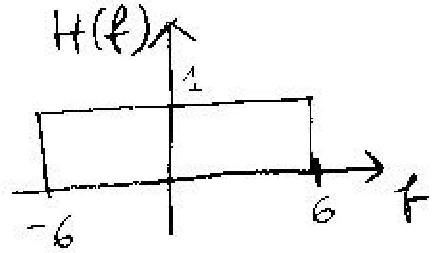
$= 8\cos(8\pi t) + 0.8 + 0.8\cos(16\pi t)$

b. $Y(f) = 0.8\delta(f) + 4\,\delta(f-4) + 4\,\delta(f+4)$

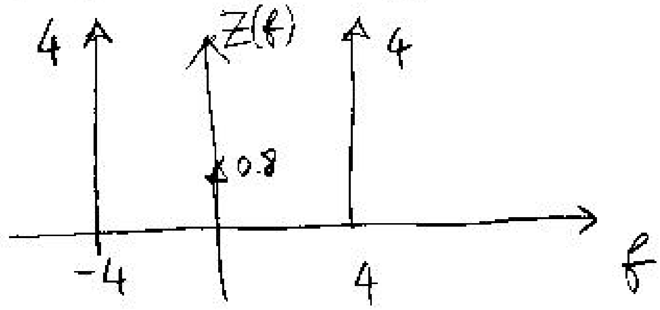
$+ 0.4\,\delta(f-8) + 0.4\,\delta(f+8)$



c. $h(t) = 12\,\text{sinc}(12t) \Longrightarrow$



Output: $Z(f) = H(f)\,Y(f)$



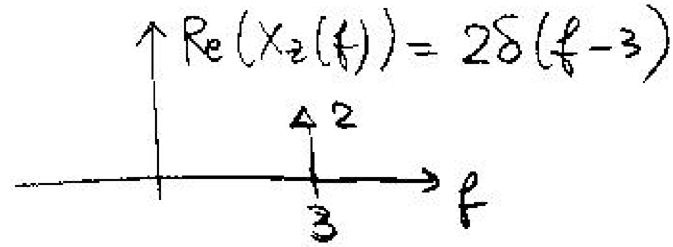
$\Longrightarrow z(t) = 0.8 + 8\cos(8\pi t)$

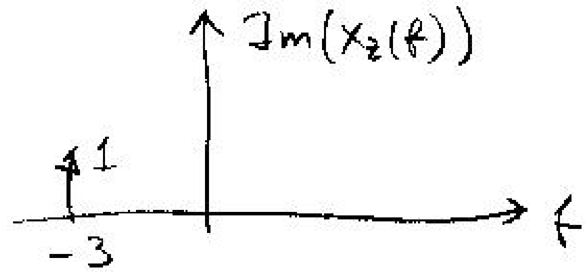
2) $x_z(t) = 2e^{j6\pi t} + j e^{-j6\pi t}$

a. $x_z(t) = 2\cos(6\pi t) + 2j\sin(6\pi t) + j\cos(6\pi t) + j(-j\sin(6\pi t))$

$$= \underbrace{\left[2\cos(6\pi t) + \sin(6\pi t)\right]}_{= x_I(t)} + j\underbrace{\left[2\sin(6\pi t) + \cos(6\pi t)\right]}_{= x_Q(t)}$$

b. $X_z(f) = 2\delta(f-3) + j\,\delta(f+3)$





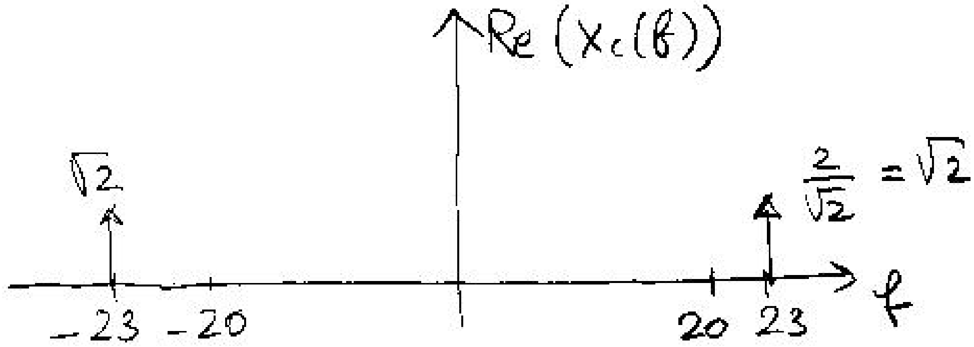
Lack of Hermitian symmetry (complex function in time domain)

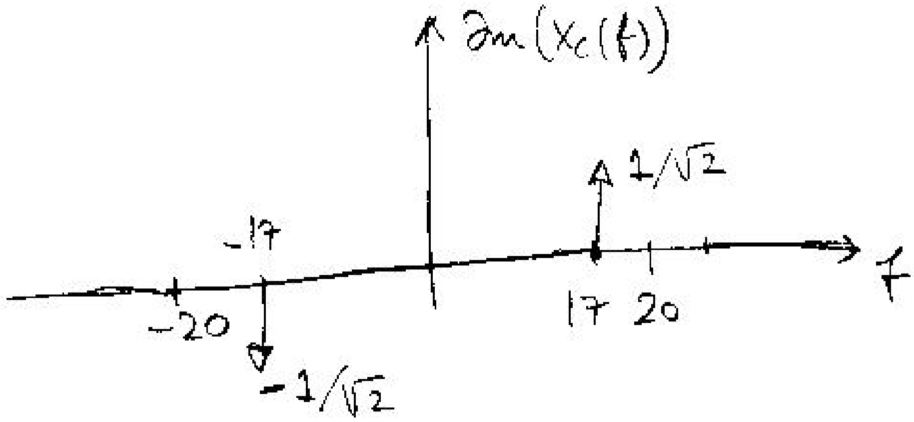
c. $x_c(t) = \sqrt{2}\,\mathrm{Re}\{x_z(t)\, e^{j2\pi\cdot 20t}\} =$

$$= \sqrt{2}\,\mathrm{Re}\{2e^{j46\pi t} + j e^{j34\pi t}\}$$

$$= \sqrt{2}\left(2\cos(46\pi t) - \sin(34\pi t)\right)$$

d. $$X_c(f) = \frac{X_z(f-f_c)}{\sqrt{2}} + \frac{X_z^*(-f-f_c)}{\sqrt{2}}$$

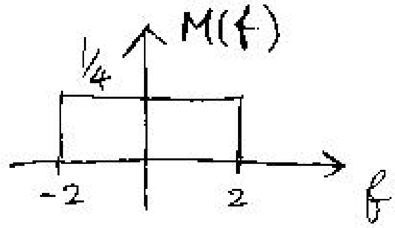


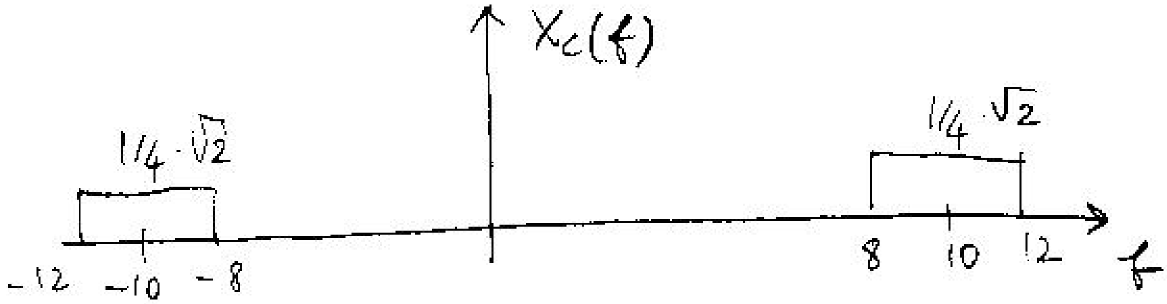


$B_T = 6\,Hz$

3. $m(t) = \text{sinc}(4t)$

a. $x_c(t) = 2\sqrt{2}\,\text{sinc}(4t)\cos(2\pi \cdot 10t)$





A good sampling frequency is $f_s = 2 \cdot 20$ Hz
$= 40$ Hz
(has to be $\geq 2 \times 12$ Hz $= 24$ Hz)

For MATLAB code, please see assignments.

4. $m(t) = \sin(2\pi t)$, $W = 1\,Hz$

a. PM: $x_c(t) = 2\sqrt{2}\cos\left(20\pi t + \sin(2\pi t)\right)$

b. Using Carson's rule

$$B_T = 2(D+1)W$$

and

$$D = \frac{k_p \max\left|\frac{d}{dt} m(t)\right|}{2\pi W} = \frac{2\pi}{2\pi W} = \frac{1}{W} = 1$$

$$\Rightarrow B_T = 2(1+1)\cdot 1 = 4\ Hz$$

c. FM: $$x_c(t) = 2\sqrt{2}\cos\left(20\pi t + \int_{-\infty}^{t} \sin(2\pi\lambda)\,d\lambda\right)$$

$$= 2\sqrt{2}\cos\left(20\pi t - \frac{1}{2\pi}\cos(2\pi t)\right)$$

d. Using Carson's rule

$$D = \frac{k_f \max|m(t)|}{W} = \frac{1}{W} = 1$$

$$\Rightarrow B_T = 2(1+1)\cdot 1 = 4\ Hz$$

5. $y_c(t) = 0.1 \cos(2\pi(t-0.1)) \cos(2000\pi(t-0.1))$

a. $y_c(t) = \sqrt{2}\, \mathrm{Re}\left\{ \frac{0.1}{\sqrt{2}} \cos(2\pi(t-0.1))\, e^{j2000\pi(t-0.1)} \right\}$

$= \sqrt{2}\, \mathrm{Re}\left\{ \underbrace{\frac{0.1}{\sqrt{2}} \cos(2\pi(t-0.1))\, e^{-j200\pi}} e^{j2000\pi t} \right\}$

$y_z(t) = \frac{0.1}{\sqrt{2}} \cos(2\pi(t-0.1))$

b.