ECE 642 - Midterm Spring 2017

Please justify all your responses (responses without justifications will not be considered). Please label your axes and plot with care.

1. (5 points) Consider an analog message $m(t)$ with Fourier transform $M(f) = P(f-1) + P(f+1)$, where $P(f)$ is a triangle centered at $f = 0$ with height 1 and base spanning the interval $[-1, 1]$ (that is, $P(f) = 0$ for $|f| > 1$ and $P(f) = 1 - |f|$ otherwise).

a. Assuming DSB-AM with carrier frequency $f_c = 100$ Hz and $A = 1$, plot the Fourier transform of the modulated signal $x_c(t)$.

b. Provide the frequency response $H(f)$ of a filter that can be applied to $x_c(t)$ to ensure spectral efficiency equal to 100%.

c. Denoting as $y_c(t)$ the output of the filter designed at step b., plot the Fourier transform $Y_z(f)$ of the equivalent baseband signal $y_z(t)$. Is $y_z(t)$ a real or a complex function?

d. Compute $y_z(t)$ (Hint: The upconversion property should be useful here. Also, recall that we have seen before what the Fourier transform of a triangle is).

2. (5 points) Assume that the same analog message $m(t) = \cos(2\pi t)$ is modulated on both in-phase and quadrature components of a passband signal, so that we have $x_I(t) = x_Q(t) = m(t)$.

a. Plot the evolution of $x_z(t)$ in the complex plane for t in the interval [0,1].

b. Compute and plot $x_A(t)$.

c. Compute and plot $x_P(t)$.

d. If the passband signal $x_c(t) = \sqrt{2}\text{Re}\{x_z(t)\exp(j2\pi f_c t)\}$ is multiplied by $\cos(2\pi f_c t - \pi/4)$ and then a low-pass filter is applied, what is the output? (Hint: $\cos(\pi/4) = \sin(\pi/4)$).

3. (2 points) Given the analog message $m(t) = 3\text{sinc}(4t)$, compute the (approximate) transmission bandwidth for FM modulation with $K_f = 1$, $A = 1$ and $f_c = 1$ GHz.