ECE 776 - Final Spring 2013

1. (2 points) We are given the periodic sequence $x^n = (0101010101...)$ with n very large.

a. Use the compression strategy that compresses by counting the number of ones and then indexing all the sequences with the same number of ones. What is the required rate?

b. Comment on the result at the previous point in light of the universality properties of the scheme used.

c. How much rate is needed by LZ77 (assume that there are no limitations on the window size)?

d. Why is LZ77 a better strategy?

2. (1 point) Consider a binary channel with $p_{Y|X}(0|0) = 1$ and $p_{Y|X}(0|1) = 1/2$. We construct a random codebook with $p_X(0) = 1/2$. What is the maximum rate that guarantees a probability of error that vanishes as the block length grows ($n \to \infty$)?

3. (2 points) Consider a BSC with bit flip probability $p = 0.2$.

a. Construct a codebook with $n = 3$ and two codewords (i.e., rate $R = 1/3$ bits/ channel use) that minimizes the probability of error.

b. Calculate the corresponding probability of error.

c. Repeat with $n = 3$ and rate $R = 2/3$ bits/ channel use (Hint: To calculate the probability of error describe an optimal decoder explicitly by giving the mapping between received signal and decoded message).

4. (1 point) Consider a BEC with bit erasure probability $\epsilon = 0.2$. For the codebook designed at point 3.a, calculate the probability of error.

5. (2 points) Consider an undirected graph with m edges. We want to show that we can always partition the vertices into two disjoint sets A and B, such that at least $m/2$ edges connect vertices in A with vertices in B. To this end, we use the probabilistic method and generate sets A and B randomly by assigning each vertex to one of the sets with equal probability and independently. Please complete the argument (Hint: You need to calculate an average...).

6. (2 points) We are given a continuous memoryless channel $Y = X + Z$, where Z is exponentially distributed with mean μ.

a. Calculate the capacity of the channel.

b. Calculate the capacity of the channel under the cost constraint $1/n \sum_{i=1}^{n} E[X_i] \leq \lambda$.

7. (1 point) For the channel $Y = HX + Z$ with H and Z independent of each other and of X, show that $I(X;Y|H) \geq I(X;Y)$. Use this result to argue that the capacity is increased if the channel H is known to the decoder.

8. (1 points) What is the minimum bandwidth ratio required to communicate a binary source $Ber(1/3)$ over a BEC with erasure probability 0.9?