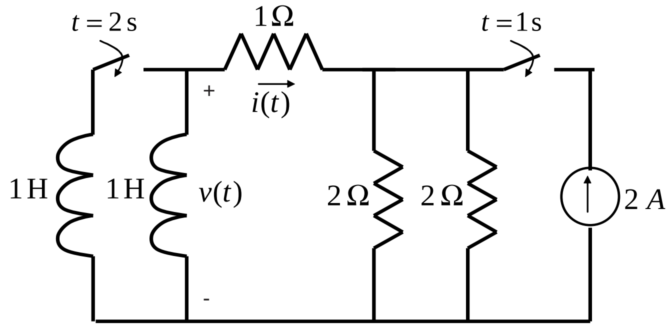
ECE 232 - Circuits and Systems II

Midterm

Please provide clear and complete answers. Don't forget to specify the units of measure!

1. Consider the circuit in the figure below. The energy stored in the inductors at time $t = 0$ is zero. The two switches are initially open, and close at the times indicated in the figure.



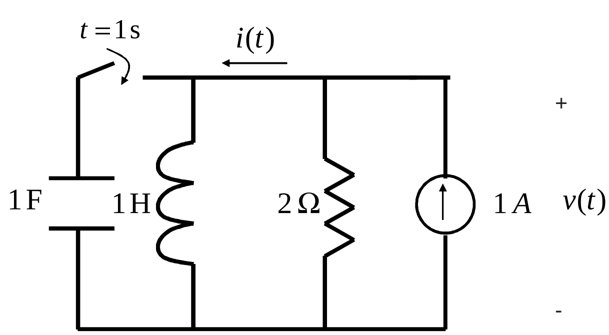
a. Calculate $i(t)$ for $0 \leq t \leq 1$.
b. Calculate $i(t)$ for $1 \leq t \leq 2$.
c. Calculate $i(t)$ for $t \geq 2$.
d. Plot $i(t)$ for $t \geq 0$.
e. Calculate and plot $v(t)$ for $t \geq 0$.

2. a.Calculate the Laplace transform of

$$(5 + e^{-4(t-1)}) \cos(3(t-1))u(t-1).$$

Please detail the properties used in your derivation.
b. Calculate and plot (in the complex plane) the poles.

3. Consider the circuit in the figure below. The energy stored in the inductor and capacitor at time $t = 0$ is zero. The switch is initially open, and closes at the times indicated in the figure.



a. Calculate $v(t)$ for $0 \leq t \leq 1$.
b. Calculate $v(t)$ for $t \geq 1$.

c. Plot $v(t)$ for $t \geq 0$.

Sol.: a. For $0 \leq t \leq 1$, we have an RL circuit with initial current equal to zero and no source. Therefore, we can calculate

$$i(t) = 0 \text{ A}$$

for $0 \leq t \leq 1$.
b. For $1 \leq t \leq 2$, we have an RL circuit with voltage source equal to 2 V (Thevenin equivalent), $L = 1$ H and $R_{eq} = 1 + 2||2 = 2\ \Omega$, and thus time constant $\tau = L/R_{eq} = 1/2$ s. Since the initial current is $i(1) = 0$ and the final value is $i(\infty) = 1$ A, we have

$$\begin{aligned} i(t) = &+1 + (0-1)e^{-2(t-1)} = \\ &+1 - e^{-2(t-1)} \text{ A} \end{aligned}$$

for $1 \leq t \leq 2$.
c. For $t \geq 2$, we have an equivalent inductor with inductance $L_{eq} = 1/2$ H, a time constant $\tau = L_{eq}/R_{eq} = 1/4$ s, initial current $i(2) = -1 - e^{-2} = +0.865$ A and final current $i(\infty) = 1$ A. It follows that

$$\begin{aligned} i(t) &= +1 + (+1 - e^{-2} - 1)e^{-4(t-2)} = \\ &= +1 - 0.135e^{-4(t-2)} \text{ A} \end{aligned}$$

for $t \geq 2$.
d. Please see notes.
e. For $0 \leq t \leq 1$, we have

$$v(t) = R_{eq} \cdot i(t) = 0 \text{ V}.$$

For $1 \leq t \leq 2$, we have

$$\begin{aligned} v(t) &= L\frac{di}{dt} \\ &= 2e^{-2(t-1)} \text{ V}. \end{aligned}$$

For $t \geq 2$, we have

$$\begin{aligned} v(t) &= L_{eq}\frac{di}{dt} \\ &= -0.27e^{-4(t-2)} \text{ V}. \end{aligned}$$

2. a. We have

$$\begin{aligned} &\mathcal{L}\{(5 + e^{-4(t-1)})\cos(3(t-1))u(t-1)\} \\ =&\mathcal{L}\{e^{-4(t-1)}\cos(3(t-1))u(t-1)\} + \mathcal{L}\{5\cos(3(t-1))u(t-1)\} \\ =&e^{-s}\mathcal{L}\{e^{-4t}\cos(3t)u(t)\} + 5e^{-s}\mathcal{L}\{\cos(3t)u(t)\} \\ =&e^{-s}\frac{(s+4)}{(s+4)^2+9} + 5e^{-s}\frac{s}{s^2+9}. \end{aligned}$$

b. The poles are given by the roots of

$$s^2 + 9 = 0,$$

namely

$$s = \pm j3,$$

and of

$$(s+4)^2 + 9 = 0,$$

namely

$$s = -4 \pm j3.$$

3. a. For $0 \leq t \leq 1$, we have an RL circuit with $\tau = L/R = 1/2$ s. We also have $i(0) = 0$ A, $i(\infty) = 1$ A and thus

$$i(t) = 1 - e^{-2t} \text{ A}.$$

It follows that

$$v(t) = \frac{di}{dt} = 2e^{-2t} \text{ V}$$

for $0 \leq t \leq 1$.
b. For $t \geq 1$, we have a parallel RLC circuit with

$$\alpha = 1/(2RC) = 1/4 \text{rad/s}$$
$$\omega_0 = 1/\sqrt{LC} = 1 \text{rad/s}.$$

The system thus operates in the underdamped regime with damped frequency $\omega_d = \sqrt{1 - 1/16} = \sqrt{15}/4 = 0.968$ rad/s. Moreover, the initial condition is $v(1) = 0$ V, since the capacitor is initially discharged, and the final value is $v(\infty) = 0$ V. It follows that

$$v(t) = e^{-\frac{1}{4}(t-1)}(B_1 \cos(0.968(t-1)) + B_2 \sin(0.968(t-1))) \text{ V},$$

where

$$B_1 = 0$$

and

$$-\frac{1}{4}B_1 + 0.968B_2 = i_c(1^+).$$

To calculate $i_c(1^+)$, we write

$$i_c(1^+) = 1 - (1 - e^{-2})$$
$$= e^{-2} = 0.135 \text{ A},$$

where we have used the fact that the voltage across the capacitor and the current in the inductor are continuous. We obtain

$$B_2 = 0.14$$

which gives us

$$v(t) = e^{-\frac{1}{4}(t-1)}(0.14 \sin(0.968(t-1))) \text{ V},$$

for $t \geq 1$.