ECE 788: Network Information Theory
Assignment 10 (due on Nov. 23)

1. Consider the block-Markov coding strategy for Decode-and-Forward (DF) discussed in class. We want to show that a backward decoding scheme would work as well as the sliding window strategy seen in class. To this end:
1.a. Start from the signal received in the last block $y^n_{3,B+1}$ and propose a decoder of w_B. Show that if $R \leq I(X_1X_2;Y)$ this decoding step has vanishing probability of error.
1.b. Argue that this can be repeated for the $w_{B-1}, w_{B-2}, \ldots$

2. In the derivation of the cut-set bound and of the DF achievable rate, we have assumed that the relay is full-duplex. Here is a way to include the half-duplex constraint (i.e., the relay cannot transmit and receive at the same time). Enlarge the input alphabet for the relay as $X_2 = [\bar{X}_2\ M_2]$, where $\bar{X}_2$ is real and $M_2 \in \{T, R\}$, where T stands for transmit mode and R for receive mode. We force $\bar{X}_2 = 0$ if $M_2 = R$. Consider the Gaussian relay channel

$$\begin{aligned} Y_2 &= \begin{cases} \frac{X_1}{d} + Z_2 & \text{if } M_2 = R \\ 0 & \text{if } M_2 = T \end{cases} \\ Y_3 &= X_1 + \frac{\bar{X}_2}{1-d} + Z_3, \end{aligned}$$

with unit-power Gaussian noise and d being the source-relay distance. Assume the standard power constraints for source and relay.
2.1. Suppose at first that the sequences of modes M_2 is fixed beforehand (independently of the messages) and that it is known to all nodes. Argue that this rate is achievable via DF

$$R_{fixed} = \min\{I(X_1;Y_2|\bar{X}_2M_2), I(X_1\bar{X}_2;Y_3|M_2)\} \tag{1}$$

for some joint distribution $P_{M_2X_1X_2}$ (Hint: just point to the necessary changes in the DF achievability proof).
2.2. From the result above, show that the following is achievable

$$R_{fixed} = \min \left\{ \begin{array}{l} P_{M_2}(R) \cdot \frac{1}{2}\log_2\left(1 + \frac{P_1}{d^2}\right), \\ P_{M_2}(R) \cdot \frac{1}{2}\log_2\left(1 + P_1\right) \\ +P_{M_2}(T) \cdot \frac{1}{2}\log_2\left(1 + P_1 + \frac{P_2}{(1-d)^2} + 2\rho\frac{\sqrt{P_1P_2}}{|1-d|}\right) \end{array} \right\}$$

(Hint: Choose appropriate random variables in (1)).
2.3. How can we improve on R_{fixed} by still assuming that the receiver knows the sequence of M_2 in advance? (Hint: Have we use the power in the most efficient way in the scheme at 2.2?)
2.4. Assume now that the receiver does not know in advance the mode M_2. Write the achievable rate with DF for such scenario in a way similar to (1). Is this rate larger or smaller than (1)? Why? (You can try to calculate it if you feel adventurous).