ECE 788: Network Information Theory
Assignment 11 (due on Nov. 30)

1. A primitive relay channel is a special relay channel defined by a pmf that factorizes as $P_{Y_2Y_3|X_1X_2} = P_{Y_2Y_{31}|X_1}P_{Y_{32}|X_2}$ where $Y_3 = [Y_{31}Y_{32}]$. In other words, the relay-destination channel $P_{Y_{32}|X_2}$ is orthogonal to the source-relay/destination channel (recall discussion in class).

1.1. Using **only** the definition above, derive the cut-set bound for this channel by starting from the general cut-set bound for a relay channel (Hint: Substitute $Y_3 = [Y_{31}Y_{32}]$ in the general cut-set bound, use the chain rule, and argue that $P_{X_1X_2} = P_{X_1}P_{X_2}$ without loss of optimality). The result should contain the capacity of the relay-destination link $R_0 = \max_{P_{X_2}} I(X_2; Y_{32})$.

1.2. Similar to the point above, using only the definition of the channel and the general expression of the rate achievable by DF, write the achievable rate by DF for the primitive relay channel. Find conditions under which DF is optimal (Hint: Define $P^*_{X_1}P^*_{X_2}$ as a distribution maximizing the cut-set bound, the cut-set bound may then equal two different expressions, ...)

1.3. Repeat the point above for CF. You should find that for R_0 large enough, CF is optimal. What is the intuition behind this result?

2. Justify all the steps in the converse of the GP theorem on pp. 69-70. In particular, why do the defined random variables satisfy the required conditions in terms of joint distribution?

3. The cut-set bound states that, for an arbitrary network with M messages, T terminals, unicast and multicast traffic (see discussion in class), the sum of the rates of the messages originating at one of the nodes in a subset $\mathcal{S}$ and destined to at least one node in the complement $\bar{\mathcal{S}}$ is limited by

$$\sum_{m \in \mathcal{M}(\mathcal{S})} R_m \leq I(X_{\mathcal{S}}; Y_{\bar{\mathcal{S}}}|X_{\bar{\mathcal{S}}}) \quad (1)$$

for some joint distribution P_{X^T} (i.e., $P_{X_1X_2\ldots X_T}$). We have defined $\mathcal{M}(\mathcal{S})$ as the indices of the messages originating at one of the nodes in $\mathcal{S}$. An outer bound on the rates $(R_1, R_2, ..., R_M)$ can then be found by fixing a P_{X^T} and considering the intersection of conditions (1) for all cuts ($\mathcal{S}$,$\bar{\mathcal{S}}$), and then taking the union of the resulting regions over all possible P_{X^T}.

3.1 Evaluate the cut-set bound for a relay channel starting from (1) and the discussion above. Check that it coincides with the one we studied in class.

3.2. Evaluate the cut-set bound for a broadcast channel with common e private messages (Hint: There are three cuts). Is this bound useful to get some capacity results? (Hint: Check the conditions on the common message)

3.3. Evaluate the cut-set bound for a multiple access channel (Hint: There are three cuts). Does it coincide with the capacity region of the multiple access channel? What is the problem, if any?