ECE 788: Network Information Theory
Assignment 6 (due on Oct. 26)

1. An encoder wants to convey an integer number $x \in \{0, 1, 2, ...\}$ to a decoder. The decoder knows a number $y \in \{0, 1, 2, ...\}$ which satisfies $0 \le x - y \le M$ for some integer M.
1.a. Consider the coding scheme $w = x \bmod (M+1)$, where w is the message sent by the encoder (mod is the usual modulo operation). How many bits R does the encoder need to convey with this strategy?
1.b. Recognize that the encoder is performing binning on the alphabet of x. Sketch the bins for $M = 1$ and $M = 2$.
1.c. Argue that, given y, the receiver will be able to recover x (you can illustrate this with an example for $M = 1$ or $M = 2$).
1.d. Suppose now y were known at the encoder too. How many bits would the encoder need to convey in this case? Is it less than with the strategy above (that assumes y only known at the decoder)?

2. Consider the Slepian-Wolf setting. The source X^n is a BSS. The side information Y^n is the output of an erasure channel with erasure probability e and input X^n.
2.a. Assume for a moment that Y^n is also known at the encoder. What would be the minimum rate required for the encoder to convey X^n losslessly to the decoder? Propose a simple scheme to achieve this rate.
2.b. What is the minimum rate required for the encoder to communicate X^n to the decoder in the Slepian-Wolf setting (where Y^n is not known at the encoder)?
2.c. Justify the result at the point above by relating it to the capacity of an erasure channel.

3. The Wyner-Ziv rate-distortion trade-off is given by

$$R_{WZ}(D) = \min_{P_{U|X},\ f(u,y)} I(U;X) - I(U;Y)$$

with $E[d(X, \hat{X})] \le D$ and $\hat{X} = f(U, Y)$.
3.a. Argue that if $D = 0$, $R_{WZ}(D) = H(X|Y)$.
3.b. Argue that if X and Y are independent, then $R_{WZ}(D) = \min_{P_{\hat{X}|X}} I(\hat{X}; X)$ with $E[d(X, \hat{X})] \le D$.
3.c. Justify the rate expression by drawing the block diagram of the achievable scheme and linking $I(U;X)$ with the error probability of the rate-distortion block (quantization) and $I(U;Y)$ to the decoding of the binning block operation (channel decoding).