ECE 232 - Circuits and Systems II
Test 1

Please provide clear and complete answers. Don't forget to specify the units of measure!

In the circuit shown in the figure, the first switch closes at time $t = 0$ and the second closes at time 2 ms. Before time $t = 0$, the circuit was in the same condition for a long time.

a. Find the current in the inductor, $i_L(t)$, for $0 \leq t \leq 2$ ms.
b. Find the current in the inductor, $i_L(t)$, for $t \geq 2$ ms (Hint: it may be useful to remember that the Thevenin equivalent of a current source of current I_S in parallel with a resistance R_S is given by a voltage source of voltage $R_S I_S$ in series with a resistor of resistance R_S).
c. Plot $i_L(t)$ for $0 \leq t \leq 4$ ms.
d. Find the current $i_0(t)$ (see figure) for $t \geq -1$ ms.
e. In the period $0 \leq t \leq 2$ ms, how much overall energy is dissipated by the series of resistors of resistance 4 Ω and 6 Ω that are in series with the inductor?

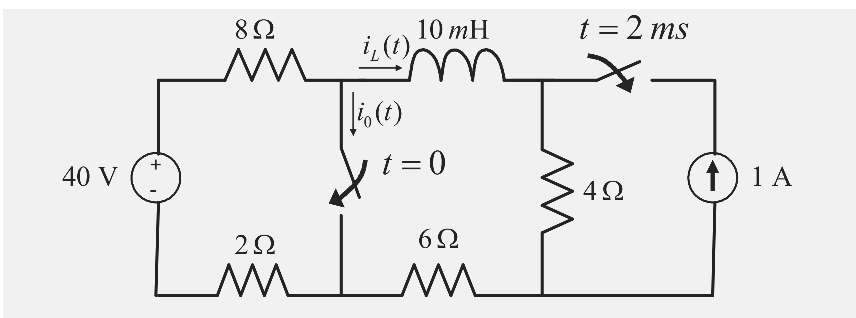

Figure 1:

Sol.:
a. For $t = 0^-$, the inductor behaves as a short circuit, and from Ohm's law, we have

$$i_L(0^-) = \frac{40}{8+2+6+4} = 2\ A.$$

And, by continuity, we also have $i_L(0^-) = i_L(0^+) = i_L(0) = 2$ A.
For $0 < t < 2$ ms, in order to evaluate current $i_L(t)$, it is sufficient to focus on the RL circuit with equivalent resistance $R_{eq} = 6 + 4 = 10$ Ω, and hence time constant $\tau = L/R_{eq} = 1$ ms. This is because the part of the circuit that includes the voltage source is in parallel to a short circuit and thus does not contribute to the current $i_L(t)$ (see also problems in the assignments). Therefore, we can write

$$i_L(t) = i_L(0)e^{-t/\tau} = 2e^{-1000t}\ A$$

for $0 \leq t \leq 2\ ms$, where equality for $t = 0$ and $t = 2\ ms$ follows by continuity. Note that $i_L(\infty) = 0$.

b. For $t \geq 2\ ms$, we transform the current source into a voltage source using the Thevenin equivalent with resistance $R_{Th} = 4\ \Omega$ and voltage source $V_{Th} = 4\ V$. This way, we obtain that $i_L(t)$ can be calculated from a RL circuit with $R_{eq} = 6 + 4 = 10\ \Omega$ and voltage source $V_{Th} = 4\ V$. Note that the time constant is still $\tau = L/R_{eq} = 1\ ms$, but

$$i_L(\infty) = -\frac{4}{10} = -0.4\ A.$$

We can then calculate

$$\begin{aligned} i_L(t) &= i_L(\infty) + (i_L(2 \cdot 10^{-3}) - i_L(\infty))e^{-1000(t-2 \cdot 10^{-3})} \\ &= -0.4 + (0.27 + 0.4)e^{-1000(t-2 \cdot 10^{-3})} \\ &= -0.4 + 0.67e^{-1000(t-2 \cdot 10^{-3})}\ A \end{aligned}$$

where we have used the continuity of the current in the inductor to obtain $i_L(2 \cdot 10^{-3}) = 2e^{-2} = 0.27\ A$.

d. For $-1\ ms \leq t \leq 0$, we clearly have $i_0(t) = 0$. For all other times, we have

$$i_0(t) = 4 - i_L(t),$$

where $i_L(t)$ was calculated above.

e. The initial energy stored in the inductor is

$$w(0) = \frac{1}{2} L i_L(0)^2 = 5 \cdot 10^{-3} \cdot 4 = 20\ mJ,$$

while at $t = 2ms$ the remaining energy is

$$w(2 \cdot 10^{-3}) = \frac{1}{2} L i_L(2 \cdot 10^{-3})^2 = 0.36\ mJ.$$

The energy that is lost, namely $20 - 0.36\ mJ = 19.64\ mJ$, has been dissipated by the two resistors at hand.