ECE 232 - Circuits and Systems II
Test 1

Please provide clear and complete answers. Don't forget to specify the units of measure!

Consider the circuit in the figure. The first switch opens at time $t = 0$ s and the second closes at time 1 s. Before time $t = 0$ s, the circuit was in the same configuration for a long time.

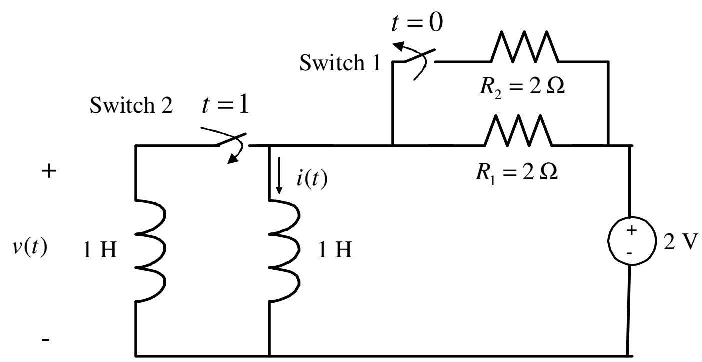

Figure 1:

a. (2 points) Find $i(0)$.

Sol.: We have $i(0) = 2/(R_1||R_2) = 2$ A.

b. (2 points) Find $i(t)$ in the interval $0 \leq t \leq 1$ s.

Sol.: In this interval, we have $i(\infty) = 2/R_1 = 1$ A and $\tau = L/R_1 = 1/2$, and hence we obtain

$$\begin{aligned} i(t) &= 1 + (2-1)e^{-2t} \\ &= 1 + e^{-2t} \text{ A.} \end{aligned}$$

c. (2 points) Plot $i(t)$ in the interval $0 \leq t \leq 1$ s.

Sol.: Please see book or class notes for the usual procedure.

d. (2 points) Calculate the energy dissipated by resistor R_1 in the interval $0 \leq t \leq 1$ s.

Sol.: The energy at hand is given by

$$\begin{aligned}
E_{R_1} &= \int_0^1 R_1 i(t)^2 dt = 2\int_0^1 (1+e^{-2t})^2 dt \\
&= 2\int_0^1 (1+e^{-4t}+2e^{-2t})dt \\
&= 2+\frac{2}{-4}(e^{-4}-1)+\frac{4}{-2}(e^{-2}-1) \\
&= 2-\frac{1}{2}(e^{-4}-1)-2(e^{-2}-1) \\
&= \frac{9}{2}-\frac{1}{2}e^{-4}-2e^{-2} = 4.22 \text{ J}.
\end{aligned}$$

e. (2 points) Evaluate the voltage $v(t)$ for $t \geq 1$ s (note that the initial current in the inductor on the left is zero).

Sol.: The equivalent inductor has inductance $L_{eq} = (L_1||L_2) = 1/2$ H and thus we have $\tau = 1/4$ s. Therefore, we get that the current flowing in the equivalent inductor is

$$i_{eq}(t) = i_{eq}(\infty) + (i_{eq}(1) - i_{eq}(\infty))e^{-4(t-1)}.$$

We can calculate $i_{eq}(\infty) = 2/2 = 1$ A and $i_{eq}(1) = i(1) = 1 + e^{-2}$ A, and thus

$$\begin{aligned}
i_{eq}(t) &= 1+e^{-2}e^{-4(t-1)} \\
&= 1+e^{-4(t-1)-2} \\
&= 1+e^{-4(t-1/2)} \text{ A}.
\end{aligned}$$

The voltage is then obtained as

$$v(t) = L_{eq}\frac{di_{eq}(t)}{dt} = -2e^{-4(t-1/2)}.$$