ECE 232 - Circuits and Systems II
Test 1

Please provide clear and complete answers. Don't forget to specify the units of measure!

Consider the circuit in Fig. 1. The switch was in position a for a long time before it moves to position b at time $t = 0$. Then it moves to position c at time $t = 2\ ms$.

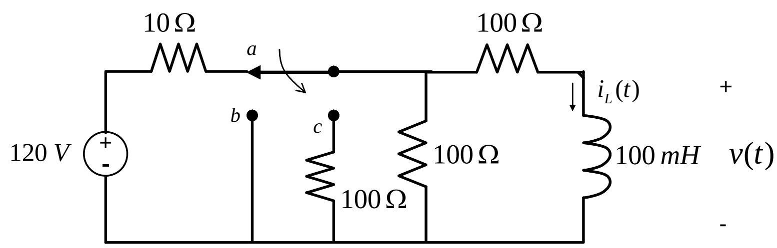

Figure 1:

a. Calculate the current $i_L(0)$.

b. Calculate $i_L(t)$ for $0 \leq t \leq 2\ ms$.

c. Calculate $i_L(t)$ for $t \geq 2\ ms$.

d. Calculate the voltage $v(t)$ across the inductors for $t \geq 0^+$.

e. Plot $v(t)$ and $i_L(t)$.

f. Calculate the energy stored on the inductor at time $t = 0$. What is the percentage of this energy that is released by the inductor in the interval $0 \leq t \leq 2\ ms$ and in the interval $t \geq 2\ ms$? Answer this question by integrating the power released by the inductor over time.

Sol.:
a. For $t = 0^-$, the current that flows on the parallel of the two 100 Ω resistors is $\frac{120}{10+50} = 2A$ so that, using the current division rule we get

$$i_L(0^-) = 2 \cdot \frac{1}{2} = 1\ A,$$

and by continuity of the current over the inductor, we get $i_L(0^-) = i_L(0^+) = i_L(0) = 1\ A$.

b. In this interval, the equivalent circuit is RL with time constant $\tau = \frac{100\times10^{-3}}{100} = 1\ ms$. The final value is zero, so that we obtain

$$i_L(t) = e^{-1000t},\ \text{for } 0 \leq t \leq 2\ ms.$$

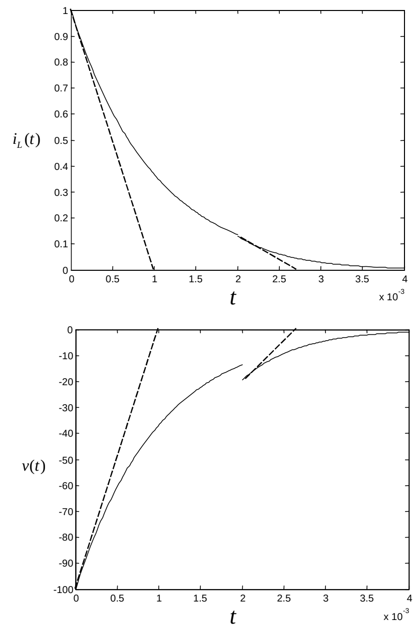

Figure 2:

c.In this interval, the circuit is RL with time constant $\tau = \frac{100\times 10^{-3}}{150} = \frac{2}{3}\ ms$. The initial value is

$$i_L(2 \times 10^{-3}) = e^{-2} = 0.13\ A.$$

so that

$$i_L(t) = 0.13 e^{-1500(t-2\times 10^{-3})},\ \text{for } t \geq 2\ ms.$$

d. The voltage is obtained as

$$\begin{aligned} v(t) &= L\frac{di(t)}{dt} \\ &= 100 \times 10^{-3} \times (-1000 e^{-1000t}) \\ &= -100 e^{-1000t} \text{ for } 0 < t < 2\ ms \end{aligned}$$

and

$$\begin{aligned} v(t) &= L\frac{di(t)}{dt} \\ &= 100 \times 10^{-3} \times (-\frac{3000}{2}) \times \left(0.13 e^{-\frac{3000}{2}(t-2\times 10^{-3})}\right) \\ &= -19.5 e^{-1500(t-2\times 10^{-3})} \text{ for } t > 2\ ms \end{aligned}$$

e. Plots are shown in the figure. Note that the voltage is discontinuous.

f. The initial energy is $E = \frac{1}{2}Li(0)^2 = \frac{1}{2}0.1 \cdot 1 = 0.05\ J = 50\ mJ$. The amount of power dissipated in the interval $0 \leq t \leq 2\ ms$ is the negative of

$$\begin{aligned}
& \int_0^{2\times10^{-3}} v(t)i(t)dt \\
= & \int_0^{2\times10^{-3}} -100e^{-1000t} \cdot e^{-1000t}dt \\
= & -100\int_0^{2\times10^{-3}} e^{-2000t}dt \\
= & \frac{-100}{-2000}\left[e^{-2000t}\right]_0^{2\times10^{-3}} \\
= & \frac{1}{20}(e^{-4} - 1) = -0.049,
\end{aligned}$$

which is the $\frac{0.049}{0.05} = 0.98\%$ of the overall initial energy.